Abelian Groups With Isomorphic Intersection Graphs

Abstract

Let G be a group. The intersection graph $\mathcal{G}(G)$ of G is an undirected graph without loops and multiple edges defined as follows: the vertex set is the set of all proper nontrivial subgroups of G, and there is an edge between two distinct vertices X and Y if and only if $X \cap Y \neq 1$ where 1 denotes the trivial subgroup of G. It was conjectured in "[B. Zelinka, *Intersection graphs of finite abelian groups*, Czechoslovak Mathematical Journal, Volume: 25, Issue: 2, Pages: 171-174, (1975)]" that two (non cyclic) finite abelian groups with isomorphic intersection graphs are isomorphic. We study this conjecture and show that it is almost true. For any finite abelian group D let D_{nc} be the product of all noncyclic Sylow subgroups of D. Our main result is that: given any two (nontrivial) finite abelian groups A and B, their intersection graphs $\mathcal{G}(A)$ and $\mathcal{G}(B)$ are isomorphic if and only if the groups A_{nc} and B_{nc} are isomorphic, and there is a bijection between the sets of (nontrivial) cyclic Sylow subgroups of A and B satisfying a certain condition. So, in particular, two finite abelian groups with isomorphic intersection graphs will be isomorphic provided that one of the groups has no (nontrivial) cyclic Sylow subgroup. Our methods are elementary.