Extending Partial Latinized Rectangles

Sibel Özkan *

April 4, 2016

Abstract

A Latin square is an $n \times n$ array filled with n different symbols, each occurring exactly once in each row and exactly once in each column. Latin squares may also be considered as multiplication tables of quasigroups. Latin squares are closely related with finite geometries and have applications in design of experiments.

Here I will talk about a generalization of a theorem of M. Hall, Jr., that an $r \times n$ Latin rectangle on n symbols can be extended to an $n \times n$ Latin square on the same n symbols.

Let $p, n, \nu_1, \nu_2, \ldots, \nu_n$ be positive integers such that $1 \leq \nu_i \leq p$ $(1 \leq i \leq n)$ and $\sum_{i=1}^{n} \nu_i = p^2$. Call an $r \times p$ matrix on n symbols $\sigma_1, \sigma_2, \ldots, \sigma_n$ an $r \times p$ $(\nu_1, \nu_2, \ldots, \nu_n)$-*latinized rectangle* if no symbol occurs more than once in any row or column, and if the symbol σ_i occurs at most ν_i times altogether $(1 \leq i \leq n)$.

I will give a necessary and sufficient condition for an $r \times p$ $(\nu_1, \nu_2, \ldots, \nu_n)$-latinized rectangle to be extendible to a $p \times p$ $(\nu_1, \nu_2, \ldots, \nu_n)$-latinized square. The condition is a generalization of P. Hall's condition for the existence of a system of distinct representatives, and will be called Hall's $(\nu_1, \nu_2, \ldots, \nu_n)$-Constrained Condition. I will then use this result to give two further sets of necessary and sufficient conditions.

*Department of Mathematics, Gebze Technical University, Cayirova Campus, Gebze - Kocaeli, 41400 TURKEY, s.ozkan@gtu.edu.tr