NUMBER-THEORY EXERCISES, IV

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Exercise 1. Prove that the following are equivalent:

(a) Every even integer greater than 2 is the sum of two primes.
(b) Every integer greater than 5 is the sum of three primes.

Exercise 2. Infinitely many primes are congruent to -1 *modulo* 6.

Exercise 3. Find all n such that

(a) $n!$ is square;
(b) $n! + (n+1)! + (n+2)!$ is square.

Exercise 4. Determine whether $a^2 \equiv b^2 \pmod{n} \implies a \equiv b \pmod{n}$.

Exercise 5. Compute $\sum_{k=1}^{1001} k^{365} \pmod{5}$.

Exercise 6. $39 \mid 53^{103} + 103^{53}$.

Exercise 7. Solve $6^{n+2} + 7^{2n+1} \equiv x \pmod{43}$.

Exercise 8. Determine whether $a \equiv b \pmod{n} \implies c^a \equiv c^b \pmod{n}$.

Exercise 9. Determine r such that $a \equiv b \pmod{r}$ whenever $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$.

Exercise 10. Solve the system

$$\begin{cases} x \equiv 1 \pmod{17}, \\ x \equiv 8 \pmod{19}, \\ x \equiv 16 \pmod{21}. \end{cases}$$

Exercise 11. The system

$$\begin{cases} x \equiv a \mod n \\ x \equiv b \mod m \end{cases}$$

has a solution if and only if $\gcd(n, m) \mid b - a$.

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