NUMBER-THEORY EXERCISES, II.V

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Exercise 1. Prove that the nth convergent of $\sqrt{5}$ is $\dfrac{2\,\mathrm{F}_{3n+2}+\mathrm{F}_{3n+3}}{\mathrm{F}_{3n+3}}$.

Exercise 2. Verify that an order $\mathfrak{O}$ of K is in particular a lattice Λ such that $\mathfrak{O}_\Lambda = \mathfrak{O}$.

Exercise 3. Let Λ and M be lattices of K. Prove the following.

(a) $\Lambda + M$ is a lattice, and

$$\langle\alpha,\beta\rangle + \langle\gamma+\delta\rangle = \langle\alpha,\beta,\gamma,\delta\rangle.$$

(b) Addition of lattices is commutative and associative.
(c) Multiplication of lattices distributes over addition.
(d) If Λ and M belong to $\mathfrak{O}$, then $\mathfrak{O} \subseteq \mathfrak{O}_{\Lambda+M}$.
(e) If Λ and M belong to $\mathfrak{O}_K$, then $\mathfrak{O}_{\Lambda+M} = \mathfrak{O}_K$.

Exercise 4. Show that $\langle n, 1+\omega\rangle$ and $\langle 1, n\omega\rangle$ both belong to $\langle 1, n\omega\rangle$.

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