

Book I of the Elements ΣΤΟΙΧΕΙΩΝ Α Öğelerin Birinci Kitabı

Euclid ΕΥΚΛΕΙΔΟΣ Öklid

September 20, 2012

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This edition of the first book of Euclid's *Elements* was prepared for a first-year undergraduate course in the mathematics department of Mimar Sinan Fine Arts University. The text has been corrected after its use in the course in the fall of 2011.

Öklid'in *Öğeler*'inin bu baskısı Mimar Sinan Güzel Sanatlar Üniversitesi, Matematik Bölümünde bir birinci sınıf lisans dersi için hazırlanmıştır. 2010-2011 Güz döneminde bu notlar ilk defa kullanılmış ve fark edilen hatalar düzeltilmiştir.

Introduction

Layout

Book I of Euclid’s *Elements* is presented here in three parallel columns: the original Greek text in the middle column, an English translation to its left, and a Turkish translation to its right.

Euclid’s *Elements* consist of 13 books, each divided into **propositions**. Some books also have **definitions**, and Book I has also **postulates** and **common notions**. In the presentation here, the Greek text of each sentence of each proposition is broken into units so that

1. each unit will fit on one line,
2. the unit as such has a role in the sentence,
3. the units, kept in the same order, make sense when translated into English.

Each proposition of the *Elements* is accompanied by

a picture of points and lines, with most points (and some lines) labelled with letters. This picture is the **lettered diagram**. We place the diagram for each proposition *after* the words. According to Reviel Netz [12, p. 35, n. 55], this is where the diagram appeared in the original scroll, presumably so that one would know how far to unroll the scroll in order to read the proposition. The end of a proposition is not to be considered as an undignified position. Indeed, Netz judges the diagram to be a *metonym* for the proposition: something associated with the proposition that is used to stand for the proposition. (Today the *enunciation* of a proposition—see § below—would appear to be the common metonym.)

Text

We receive Euclid’s text through various filters. The *Elements* are supposed to have been composed around 300 B.C.E. Heiberg’s text (published in 1883) is based mainly on a manuscript in the Vatican written the tenth century C.E., closer to our time than to Euclid’s time. Knorr [8] argues that Euclid’s original intent may be better reflected in some Arabic translations from the eighth and ninth centuries. (The argument is summarized in [9].) Nonetheless, we shall just use the Heiberg text.

More precisely, for convenience, we take the Greek text in our underlying L^AT_EX file from the L^AT_EX files of Richard Fitzpatrick, who has published his own parallel English translation.¹ (In the underlying L^AT_EX file, the enunciation of Proposition I.1 in Greek reads as in Table 1.) Fitzpatrick reports that his Greek text is that of Heiberg,

but he gives it without Heiberg’s *apparatus criticus*. Also his method of transcription is unclear. There is at least one mistake in his text (τρὸς for πρὸς near the beginning of I.5). We shall correct such mistakes, if we find them, although we shall not look for them systematically.

In the process of translating, we have made use of a printout of the Greek text of Myungsun Ryu.² We do not have a L^AT_EX file for this text; only pdf. The text is said to be taken from the *Perseus Digital Library*.

We also refer to images of Heiberg’s original text [1], which are available as pdf files from the Wilbour Hall website³ and from European Cultural Heritage Online (ECHO).⁴ In preparing the files from the latter source for printing, we have trimmed the black borders by means of a program called **briss**.⁵

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>Ep'i t~hc doje'ishc e>uje'iac peperasm'enhc tr'igwnon >is'opleuron sust'hsasjai.
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Table 1: Greek text, coded for L^AT_EX

Analysis

Each proposition of the *Elements* can be understood as being a **problem** or a **theorem**. Writing around 320 C.E., Pappus of Alexandria [17, pp. 564–567] describes the

distinction:

Those who favor a more technical terminology in geometrical research use

¹<http://farside.ph.utexas.edu/euclid.html>

²<http://en.wikipedia.org/wiki/File:Euclid-Elements.pdf>

³<http://www.wilbourhall.org/>

⁴<http://echo.mpiwg-berlin.mpg.de/home/>

⁵<http://briss.sourceforge.net/>

⁶Ivor Thomas [17, p. 567] uses *inquiry* here in his translation; but there is *no* word in the Greek original corresponding to this or to *proposition*.

- **problem** (πρόβλημα) to mean a [proposition⁶] in which it is proposed to do or construct [something]; and
- **theorem** (θεώρημα), a [proposition] in which the consequences and necessary implications of certain hypotheses are investigated;

but among the ancients some described them all as problems, some as theorems.

In short, a problem proposes something to *do*; a theorem proposes something to *see*. (The Greek for *theorem* means more generally ‘that which is looked at’ and is related to the verb θεάομαι ‘look at’; from this also comes θέατρον ‘theater’.)

Be it a problem or a theorem, a proposition—or more precisely the *text* of a proposition—can be analyzed into as many as six parts. The Green Lion edition [3, p. xxiii] of Heath’s translation of Euclid describes this analysis as found in Proclus’s *Commentary on the First Book of Euclid’s Elements* [14, p. 159]. In the fifth century C.E., Proclus⁷ writes:

Every problem and every theorem that is furnished with all its parts should contain the following elements:

- 1) an **enunciation** (πρότασις),
- 2) an **exposition** (ἔκθεσις),
- 3) a **specification** (διορισμός),
- 4) a **construction** (κατασκευή),
- 5) a **proof** (ἀπόδειξις), and
- 6) a **conclusion** (συμπέρασμα).

Of these, the enunciation states what is given and what is being sought from it, for a perfect enunciation consists of both these parts. The exposition takes separately what is given and prepares it in advance for use in the investigation. The specification takes separately the thing that is sought and makes clear precisely what it is. The construction adds what is lacking in the given for finding what is sought. The proof draws the proposed inference by reasoning scientifically from the propositions that have been admitted. The conclusion reverts to the enunciation, confirming what has been proved.

So many are the parts of a problem or a theorem. The most essential ones, and those which are always present, are enunciation, proof, and conclusion.

Alternative translations are:

Language

The Greek language that we have begun discussing is the language of Euclid: *ancient* Greek. This language belongs to the so-called Indo-European family of languages. English also belongs to this family, but Turkish does not.

o.o.1 Writing

- for ἔκθεσις, *setting out*, and
- for διορισμός, *definition of goal* [12, p. 10].

Heiberg’s analysis of the text of the *Elements* into paragraphs does not correspond exactly to the analysis of Proclus; but Netz uses the analysis of Proclus in his *Shaping of Deduction in Greek Mathematics* [12], and we shall use it also, according to the following understanding:

1. The *enunciation* of a proposition is a general statement, without reference to the lettered diagram. The statement is about some subject, perhaps a straight line or a triangle.

2. In the *exposition*, that subject is identified in the diagram by means of letters; the existence of the subject is established by means of a third-person imperative verb.

3. (a) The *specification* of a *problem* says what will be done with the subject, and it begins with the words δεῖ δὴ. Here δεῖ is an impersonal verb with the meaning of ‘it is necessary to’ or ‘it is required to’ or simply ‘one must’; while δὴ is a ‘temporal particle’ with the root meaning of ‘at this or that point’ [10]. That which is necessary is expressed by a clause with an infinitive verb. In translating, we may use the English form ‘It is necessary for *A* to be *B*.’

(b) The specification of a *theorem* says what will be proved about the subject, and it begins with the words λέγω ὅτι ‘I say that’. The same expression may also appear in a problem, in an additional specification at the head of the proof, after the construction.

4. In the *construction*, if it is present, the second word is often γάρ, a ‘confirmatory adverb and causal conjunction’ [16, ¶2803, p. 637]. We translate it as ‘for’, at the beginning of the sentence; but again, γάρ itself is the second word, because it is *postpositive*: it simply never appears at the beginning of a sentence.

5. Then the *proof* often begins with the particle ἐπεὶ ‘because, since’. The ἐπεὶ (or other words) may be followed by οὖν, a ‘confirmatory or inferential’ postpositive particle [16, ¶2955, p. 664].

6. The *conclusion* repeats the enunciation, usually with the addition of the postpositive particle ἄρα ‘therefore’. Then, after the repeated enunciation, the conclusion ends with one of the clauses:

(a) ὅπερ ἔδει ποιῆσαι ‘just what it was necessary to do’ (in problems); Heiberg translates this into Latin as *quod oportebat fieri*, although *quod erat faciendum* or QEF is also used;

(b) ὅπερ ἔδει δεῖξαι ‘just what it was necessary to show’ (in theorems): in Latin, *quod erat demonstrandum*, or QED.

However, in some ways, Turkish is closer to Greek than English is. Modern scientific terminology, in English or Turkish, often has its origins in Greek.

⁷Proclus was born in Byzantium (that is, Constantinople, now Istanbul), but his parents were from Lycia (Likyā), and he was ed-

ucated first in Xanthus. He moved to Alexandria, then Athens, to study philosophy [14, p. xxxix].

capital	minuscule	transliteration	name
A	α	a	alpha
B	β	b	beta
Γ	γ	g	gamma
Δ	δ	d	delta
E	ε	e	epsilon
Z	ζ	z	zeta
H	η	ê	eta
Θ	θ	th	theta
I	ι	i	iota
K	κ	k	kappa
Λ	λ	l	lambda
M	μ	m	mu
N	ν	n	nu
Ξ	ξ	x	xi
O	ο	o	omicron
Π	π	p	pi
P	ρ	r	rho
Σ	σ, ς	s	sigma
T	τ	t	tau
Υ	υ	y, u	upsilon
Φ	φ	ph	phi
X	χ	ch	chi
Ψ	ψ	ps	psi
Ω	ω	ô	omega

Table 2: The Greek alphabet

The Greek alphabet, in Table 2, is the source for the Latin alphabet (which is used by English and Turkish), and it is a source for much scientific symbolism. The vowels of the Greek alphabet are α, ε, η, ι, ο, υ, and ω, where η is a long ε, and ω is a long ο; the other vowels (α, ι, υ) can be long or short. Some vowels may be given tonal accents (acute, grave, circumflex). An initial vowel takes either a rough-breathing mark (as in ἄ) or a smooth-breathing mark (ᾰ): the former mark is transliterated by a preceding h, and the latter can be ignored, as in ὑπερβολή *hyperbolê* *hyperbola*, ὀρθογώνιον *orthogōnion* *rectangle*. Likewise, ῥ is transliterated as rh, as in ῥόμβος *rhombos* *rhombus*. A long vowel may have an iota subscript (α, η, ω), especially

Nouns

As in Turkish, so in Greek, a single noun or verb can appear in many different forms. The general analysis is the same: the noun or verb can be analyzed as STEM + ENDING (*gövde + ek*).⁸

Like a Turkish noun, a Greek noun changes to show distinctions of *case* and *number*. Unlike a Turkish noun, a Greek noun does not take a separate ending (such as *-ler*) for the plural number; rather, each case-ending has a singular form and a plural form. (There is also a dual form, but this is rarely seen, although the distinction between the dual and the plural number occurs for example in ἐκάτερος/ἕκαστος ‘either/each’.)

Unlike a Turkish noun, a Greek noun has one of three

in case-endings of nouns. Of the two forms of minuscule sigma, the ς appears at the ends of words; elsewhere, σ appears, as in βᾰσις *basis* *base*.

In increasing strength, the Greek punctuation marks are [, · ·], corresponding to our [; ; ·]. (The Greek question-mark is like our semicolon, but it does not appear in Euclid.)

Euclid himself will have used only the capital letters; the minuscules were developed around the ninth century [16, ¶2, p. 8]. The accent marks were supposedly invented around 200 B.C.E., because the pronunciation of the accents was dying out [16, ¶161, p. 38].

genders: masculine, feminine, or neuter. We can use this notion to distinguish nouns that are *substantives* from nouns that are *adjectives*. A substantive always keeps the same gender, whereas an adjective *agrees* with its associated noun in case, number, and gender.⁹ (Turkish does not show such agreement.)

The Greek cases, with their rough counterparts in Turkish, are as follows:

1. nominative (the dictionary form),
2. genitive (*-in hâli* or *-den hâli*),
3. dative (*-e hâli* or *-le hâli*¹⁰ or *-de hâli*),
4. accusative (*-i hâli*),
5. vocative (usually the same as the nominative, and

⁸The stem may be further analyzable as ROOT + CHARACTERISTIC.

⁹English retains the notion of gender only in its personal pronouns: *he*, *she*, *it*. If masculine and feminine are together the *animate* genders, and neuter the *inanimate*, then the distinction be-

tween animate and inanimate is shown in *who/which*. Agreement of adjective with noun in English is shown in the demonstratives: *this word/these words*.

¹⁰One source, Özkırmlı [15, p. 155], does indeed treat *-le* as one of the *durum* or *hâl ekleri*.

anyway it is not needed in mathematics, so we shall ignore it below).

The accusative case is the case of the direct object of a verb. Turkish assigns the ending *-i* only to *definite* direct objects; otherwise, the nominative is used. However, for a neuter Greek noun, the accusative case is always the same as the nominative.¹¹

A Greek noun is of the *vowel declension* or the *consonant declension*, depending on its stem. Within the vowel declension, there is a further distinction between the *ā-* or *first declension* and the *o-* or *second declension*. Then the

consonant declension is the *third declension*. The spelling of the case of a noun depends on declension and gender. Turkish might be said to have four declensions; but the variations in the case-endings in Turkish are determined by the simple rules of vowel harmony, so that it may be more accurate to say that Turkish has only one declension. Some variations in the Greek endings are due to something like vowel harmony, but the rules are much more complicated. Some examples are in Table 3.

The meanings of the Greek cases are refined by means of *prepositions*, discussed below.

		1st feminine	1st feminine	2nd masculine	2nd neuter	3rd neuter
singular	nominative	γραμμή	γωνία	κύκλος	τρίγωνον	μέρος
	genitive	γραμμής	γωνίας	κύκλου	τριγώνου	μέρους
	dative	γραμμῇ	γωνίᾳ	κύκλῳ	τριγώνῳ	μέρει
	accusative	γραμμήν	γωνίαν	κύκλον	τρίγωνον	μέρος
plural	nominative	γραμμαί	γωνίαι	κύκλοι	τρίγωνα	μέρη
	genitive	γραμμῶν	γωνίων	κύκλων	τριγώνων	μέρων
	dative	γραμμαῖς	γωνίαις	κύκλοις	τριγώνοις	μέρεσι
	accusative	γραμμάς	γωνίας	κύκλους	τρίγωνα	μέρη
		<i>line</i>	<i>angle</i>	<i>circle</i>	<i>triangle</i>	<i>part</i>

Table 3: Declension of Greek nouns

The definite article

	m.	f.	n.
nom.	ὁ	ἡ	τό
gen.	τοῦ	τῆς	τοῦ
dat.	τῷ	τῇ	τῷ
acc.	τόν	τήν	τό
nom.	οἱ	αἱ	τά
gen.	τῶν	τῶν	τῶν
dat.	τοῖς	ταῖς	τοῖς
acc.	τούς	τάς	τά

Table 4: The Greek article

Greek has a definite article, corresponding somewhat to the English *the*. Whereas *the* has only one form, the Greek article, like an adjective, shows distinctions of gender, number, and case, with forms as in Table 4.

Euclid may use (a case-form of) τό A σημείον ‘the A point’ or ἡ AB εὐθεία [γραμμῇ] ‘the AB straight [line]’. Here the letters A and AB come between the article and the noun, in what Smyth calls *attributive* position [16, ¶1154]. Then A itself is not a point, and AB is not a line; the point and the line are seen in a diagram, *labelled* with the indicated letters. However, Euclid may omit the noun, speaking of τό A ‘the A’ or ἡ AB ‘the AB’.

Sometimes (as in Proposition 3) a single letter may denote a straight line; but then the letter takes the feminine article, as in ἡ Γ ‘the Γ’, since γραμμῇ ‘line’ is feminine. Netz [12, 3.2.3, p.113] suggests that Euclid uses the neuter

σημεῖον rather than the feminine στιγμή for ‘point’ so that points and lines will have different genders. (See Proposition 43 for a related example.)

In general, an adjective may be given an article and used as a substantive. (Compare ‘The best is the enemy of the good’, attributed to Voltaire in the French form *Le mieux est l’ennemi du bien*.¹²) The adjective need not even have the article. Euclid usually (but not always) says *straight* instead of *straight line*, and *right* instead of *right angle*. In our translation, we use STRAIGHT and RIGHT when the substantives *straight line* and *right angle* are to be understood.

Euclid may also refer (as in Proposition 5) to κοινή ἡ ΒΓ ‘the ΒΓ, which is common’. Here the adjective ‘common’ would appear to be in *predicate* position [16, ¶1168]. In this position, the adjective serves not to dis-

¹¹English nouns retain a sort of genitive case, in the possessive forms: *man/man’s/men/men’s*. There are further case-distinctions in pronouns: *he/his/him, she/her, they/their/them*.

¹²<http://en.wikiquote.org/wiki/Voltaire>, accessed July 8, 2011.

tinguish the straight line in question from other straight lines, but to express its relation to other parts of the diagram (in this case, that it is the base of two different triangles).

Similarly, Euclid may use the adjective ὅλος *whole* in predicate position, as in Proposition 4: ὅλον τὸ ABΓ τρίγωνον ἐπὶ ὅλον τὸ ΔEZ τρίγωνον ἐφαρμόσει ‘the ABΓ triangle, as a whole, to the ΔEZ triangle, as a whole, will apply’. Smyth’s examples of adjective position include:

attributive: τὸ ὅλον στράτευμα *the whole army*;

predicate: ὅλον τὸ στράτευμα *the army as a whole*.

The distinction here may be that the whole army may have attributes of a person, as in ‘The whole army is hungry’; but the army as a whole does not (as a whole, it is not a person). The distinction is subtle, and in the example

τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτείνουσας πλευρᾶς τετράγωνον
ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτείνουσας πλευρᾶς
τὴν ὀρθὴν γωνίαν

the right angle
on the side subtending the right angle
the square on the side subtending the right angle

Table 5: Nesting of Greek adjective phrases

Prepositions

In the example in Table 5, the preposition ἀπό appears. This is used only before nouns in the genitive case. It usually has the sense of the English preposition *from*, as in the first postulate, or in the construction of Proposition 1, where straight lines are drawn *from* the point Γ to A and B. In Table 5 then, the sense of the Greek is not exactly that the square sits *on* the side, but that it arises *from* the side.

Euclid uses various prepositions, which, when used before nouns in various cases, have meanings roughly as in Table 6. Details follow.

When its object is in the accusative case, the preposition ἐπὶ has the sense of the English preposition *to*, as again in the first postulate, or in the construction of Proposition 1, where straight lines are drawn from Γ *to* A and B.

The prepositional phrase ἐπὶ τὰ αὐτὰ μέρη ‘to the same parts’ is used several times, as for example in the fifth postulate and Proposition 7. The object of the preposition ἐπὶ is again in the accusative case, but is plural. It would appear that, as in English, so in Greek, ‘parts’ can have the sense of the singular ‘region’. More precisely in this case, the meaning of ‘parts’ would appear to be ‘side [of a straight line]’; and one might translate the phrase ἐπὶ τὰ αὐτὰ μέρη by ‘on the same side’ (as Heath does).¹⁴ The more general sense of ‘part’ is used in the fifth common notion.

The object of the preposition ἐπὶ may also be in the

¹³This is an elaboration of an observation by Netz [12, 3.2.1, p. 105; 4.2.1.1, pp. 133–4].

¹⁴According to Netz [12, 3.2.2, p. 112], ‘parts’ means ‘direction’ in this phrase, and only in this phrase.

from Euclid, Heath just gives the translation ‘the whole triangle’.

In Proposition 5, Euclid refers to ἡ ὑπὸ ABΓ γωνία, which perhaps stands for ἡ περιεχομένη ὑπὸ τῆς ABΓ γραμμῆς γωνία ‘the contained-by-the-ABΓ-line angle’ or ἡ περιεχομένη ὑπὸ τῶν AB, BΓ εὐθειῶν γραμμῶν γωνία ‘the bounded-by-the-AB-BΓ-straight-lines angle’.¹³ In the same proposition, the form γωνία ἡ ὑπὸ ABΓ appears (actually γωνία ἡ ὑπὸ BZΓ), with no obvious distinction in meaning. (Each position of [ἡ] ὑπὸ ABΓ is called attributive by Smyth.) For short, Euclid may say just ἡ ὑπὸ ABΓ for the angle, without using γωνία.

The nesting of adjectives between article and noun can be repeated. An extreme example is the phrase from the enunciation of Proposition 47 analyzed in Table 5.

genitive case. Then ἐπὶ has the sense of *on*, as yet again in the construction of Proposition 1, where a triangle is constructed *on* the straight line AB.

The preposition πρὸς is used in the set phrase πρὸς ὀρθᾶς [γωνίας] *at right angles*, where the noun phrase ὀρθῆ [γωνία] *right [angle]* is a plural accusative. Also in the definitions of angle and circle, πρὸς is used with the accusative, in a sense normally expressed in English by ‘to’. In every other case in Euclid’s Book I, πρὸς is used with the dative case and also has the sense of *at* or *on* as for example in Proposition 2, where a straight line is to be placed *at* a given point.

There is a set phrase, used in Propositions 14, 23, 24, 31, 42, 45, and 46, in which πρὸς appears twice: πρὸς τῆ εὐθείᾳ καὶ τῷ πρὸς αὐτῆ σημείῳ ‘*at the straight [line] and [at] the point on it*’. (It is assumed here that the *first* occurrence of πρὸς takes two objects, both STRAIGHT and *point*. It is unlikely that *point* is un-governed, since according to Smyth [16, ¶1534], in prose, ‘the dative of place (chiefly *place where*) is used only of proper names’.)

The preposition διὰ is used with the accusative case to give *explanations*. The explanation might be a clause whose verb is an infinitive and whose subject is in the accusative case itself; then the whole clause is given the accusative case by being preceded by the neuter accusative article τό.¹⁵ The first example is in Proposition 4: διὰ τὸ ἴσην εἶναι τὴν AB τῆ ΔE ‘because AB is equal to ΔE’.

The preposition διὰ is also used with the genitive case,

¹⁵It may however be pointed out that the article τό could also be in the nominative case. However, prepositions are never followed by a case that is unambiguously nominative.

with the sense of *through* as in speaking of a straight line *through* a point. This use of *διά* always occurs in a set phrase as in the enunciation of Proposition 31, where the straight line through the point is also parallel to some other straight line.

The preposition *κατά* is used in Book I always with a name or a word for a *point* in the accusative case. This point may be where two straight lines meet, as in Proposition 27, or where a straight line is bisected, as in Proposition 10. The set phrase *κατά κορυφήν* ‘at a head’ occurs for example in the enunciation of Proposition 15 to describe angles that are ‘vertically opposite’ or simply *vertical*.

The preposition *μετά*, used with the genitive case, means *with*. It occurs in Book I only in Proposition 43, only with the names of triangles, only in the sentence τὸ ΑΕΚ τρίγωνον μετὰ τοῦ ΚΗΓ ἴσον ἐστὶ τῷ ΑΘΚ τρίγωνῳ μετὰ τοῦ ΚΖΓ ‘Triangle ΑΕΚ, with [triangle] ΚΗΓ, is equal to triangle ΑΘΚ with [triangle] ΚΖΓ’.

The preposition *παρά* is used in Book I only in Proposition 44, with the name of a straight line in the genitive case; and then the preposition has the sense of *along*: a parallelogram is to be constructed, one of whose sides is set *along* the original straight line so that they coincide.

The adjective *παράλληλος* ‘parallel’, used frequently starting with Proposition 27, seems to result from *παρά* + *ἀλλήλων* ‘alongside one another’. Here *ἀλλήλων* is the reciprocal pronoun ‘one another’, never used in the singular or nominative; it seems to result from *ἄλλος* ‘another’. The dative plural *ἀλλήλοις* occurs frequently, as in Proposition 1, where circles cut *one another*, and two straight lines are equal *to one another*.

The preposition *ὕπο* is used in naming angles by letters, as in ἡ ὑπὸ ΑΒΓ γωνία ‘the angle ΑΒΓ’. Possibly such a phrase arises from a longer phrase, as in Proposition 4, ἡ γωνία ἡ ὑπὸ τῶν εὐθειῶν περιεχομένη ‘the angle that is

contained *by* the [two] sides [elsewhere indicated]’. Here *ὕπο* precedes the agent of a passive verb, and the noun for the agent is in the genitive case. There is a similar use in the enunciation of Proposition 9: ἡ ὑπὸ ΒΑΓ γωνία δίχα τέτμηται ὑπὸ τῆς ΑΖ εὐθείας ‘The angle ΒΑΓ is bisected *by* the [straight line] ΑΖ’.

The preposition *ὕπο* is also used with nouns in the accusative case. It may then have the meaning of *under*, as in Proposition 5. More commonly it just precedes objects of the verb *ὑποτείνω* ‘stretch under’, used in English in the Latinate form *subtend*. The subject of this verb will be the side of a triangle, and the object will be the opposite angle.

The preposition *ἐν* ‘in’ is used only with the dative, frequently in the phrase *ἐν ταῖς αὐταῖς παραλλήλοις* ‘in the same parallels’, starting with Proposition 35. It is used in Proposition 42 and later with reference to parallelograms *in* a given angle. Finally, in Proposition 47 (the so-called Pythagorean Theorem), there is a general reference to a situation *in* right-angled triangles.

The preposition *ἐξ* ‘from’ is used with the genitive case. In Proposition 7, in the set phrase *ἐξ ἀρχῆς* ‘from the beginning’, that is, *original*. Beyond this, *ἐξ* appears only in the problematic definitions of straight line and plane surface, in the set phrase *ἐξ ἰσού*: ‘from equality’ or, as Heath has it, ‘evenly’.

The preposition *περὶ* ‘about’ is used only in Propositions 43 and 44, only with the accusative, only with reference to figures arranged *about* the diameter of a parallelogram.

Greek has a few other prepositions: *σύν*, *ἀντί*, *πρό*, *ἀμφί*, and *ὑπέρ*; but these are not used in Book I. Any of the prepositions may be used also as a *prefix* in a noun or verb.

Verbs

A *verb* may show distinctions of *person*, *number*, *voice*, *tense*, *mood (mode)*, and *aspect*. Names for the forms that occur in Euclid are:

1. *mood*: indicative, imperative, or subjunctive;
2. *aspect*: continuous, perfect, or aorist;
3. *number*: singular or plural;
4. *voice*: active or passive;
5. *person*: first or third;
6. *tense*: past, present, or future.

(In other Greek writing there are also a *second* person, a *dual* number, and an *optative* mood. One speaks of a *middle* voice, but this usually has the same form as the passive.) Euclid also uses *verbal nouns*, namely *infinitives* (verbal substantives) and *participles* (verbal adjectives).

Suppose the utterance of a sentence involves three things: the *speaker* of the sentence, the *act* described by

the sentence, and the *performer* of the act. If only for the sake of remembering the six verb features above, one can make associations as follows:

1. mood: speaker
2. aspect: act
3. number: performer
4. voice: performer–act
5. person: speaker–performer
6. tense: act–speaker.

First-person verbs are rare in Euclid. As noted above, λέγω ‘I say’ is used at the beginning of specifications of theorems, and a few other places. Also, δείξομεν ‘we shall show’ is used a few times. The other verbs are in the third person.

Of the 48 propositions of Book I, 14 have enunciations of the form Ἴάν + SUBJUNCTIVE.

Often in sentences of the logical form ‘If *A*, then *B*’, Euclid will express ‘If *A*’ as a *genitive absolute*, a noun and participle in the genitive case. We use the corresponding absolute construction in English.

Translation

	genitive	dative	accusative
ἀπό	from		
διά	through [a point]		owing to
ἐν		in	
ἐξ	from [the beginning]		
ἐπι	on		to
κατά			at [a point]
μετά	with		
παρά	along [a straight line]		
περί			about
πρός		at/on	at [right angles]
ὑπό	by		under

Table 6: Greek prepositions

The Perseus website,¹⁶ with its Word Study Tool, is useful for parsing. However, in the work of interpreting the Greek, we also consult print resources, such as Smyth’s *Greek Grammar* [16], the *Greek-English Lexicon* of Liddell, Scott, and Jones [10], the *Pocket Oxford Classical Greek Dictionary* [11], and Heath’s translation of the *Elements* [3, 2].

There are online lessons on reading Euclid in Greek.¹⁷

In translating Euclid into English, Heath seems to stay as close to Euclid as possible, under the requirement that the translation still read well *as English*. There may be subtle ways in which Heath imposes modern ways of thinking that are foreign to Euclid.

The English translation here tries to stay even closer to Euclid than Heath does. The purpose of the translation is to elucidate the original Greek. This means the translation may not read so well as English. In particular, word order may be odd. Simple declarative sentences in English normally have the order SUBJECT-VERB-OBJECT (or SUBJECT-COPULA-PREDICATE). When Euclid uses another order, say SUBJECT-OBJECT-VERB (or SUBJECT-PREDICATE-COPULA), the translation *may* follow him. There is a precedent for such variations in English order, albeit from a few centuries ago. For example, there is the rendition by George Chapman (1559?–1634) of Homer’s *Iliad* [13]. Chapman begins his version of Homer thus:

Achilles’ banefull wrath resound, O Goddess,
that imposd
Infinite sorrowes on the Greekes, and many
brave soules losd
From breasts Heroique—sent them farre, to
that invisible cave
That no light comforts; and their lims to dogs
and vultures gave.
To all which Jove’s will gave effect; from whom
first strife begunne
Betwixt Atrides, king of men, and Thetis’ god-

like Sonne.

The word order SUBJECT-PREDICATE-COPULA is seen also in the lines of Sir Walter Raleigh (1554?–1618), quoted approvingly by Henry David Thoreau (1817–62) [18]:

But men labor under a mistake. The better part of the man is soon plowed into the soil for compost. By a seeming fate, commonly called necessity, they are employed, as it says in an old book, laying up treasures which moth and rust will corrupt and thieves break through and steal.¹⁸ It is a fool’s life, as they will find when they get to the end of it, if not before. It is said that Deucalion and Pyrrha created men by throwing stones over their heads behind them:—

*“Inde genus durum sumus, experien-
sque laborum,
Et documenta damus qua simus origine
nati.”*

Or, as Raleigh rhymes it in his sonorous way,—

*“From thence our kind hard-hearted is,
enduring pain and care,
Approving that our bodies of a stony
nature are.”*

So much for a blind obedience to a blundering oracle, throwing the stones over their heads behind them, and not seeing where they fell.¹⁹

More examples:

The man recovered of the bite,
The dog it was that died.²⁰

Whose woods these are I think I know.
His house is in the village though;
He will not see me stopping here
To watch his woods fill up with snow.²¹

¹⁶<http://www.perseus.tufts.edu/hopper/collection?collection=Perseus%3Acorpus%3Aperseus%2Cwork%2CEuclid%2C%20Elements>

¹⁷<http://www.du.edu/~etuttle/classics/nugreek/contents.htm>

¹⁸The Gospel According to St Matthew, 6:19: ‘Lay not up for yourselves treasures upon earth, where moth and rust doth corrupt, and where thieves break through and steal’.

¹⁹Text taken from <http://www.gutenberg.org/files/205/205-h/>

205-h.htm, July 6, 2011.

²⁰The last lines of ‘An Elegy on the Death of a Mad Dog’ by Oliver Goldsmith (1728–1774) (http://www.poetry-archive.com/g/an_elegy_on_the_death_of_a_mad_dog.html, accessed July 12, 2011).

²¹The first stanza of ‘Stopping by Woods on a Snowy Evening’ by Robert Frost (<http://www.poetryfoundation.org/poem/171621>, accessed July 12, 2011).

Giriş

Sayfa düzeni ve Metin

Öklid'in *Öğelerinin* birinci kitabı, burada üç sütun halinde sunuluyor: orta sütunda orijinal Yunanca metin, onun solunda bir İngilizce çevirisi ve sağında bir Türkçe çevirisi yer alıyor.

Öklid'in *Öğeleri*, her biri **önermelere** bölünmüş olan 13 kitaptan oluşur. Bazı kitaplarda **tanımlar** da vardır. Birinci kitap ayrıca **postülatları** ve **genel kavramları** da içerir. Yunanca metnin her önermesinin her cümlesi öyle birimlere bölünmüştür ki

1. her birim bir satıra sığar,
2. birimler cümle içinde bir rol oynarlar
3. İngilizceye çevirirken birimlerin sırasını korumak anlamlı olur.

Analiz

Öğelerin her önermesi bir **problem** veya bir **teorem** olarak anlaşılabilir. M.S. 320 civarında yazan İskenderiyeli Pappus bu ayrımı tarif ediyor [17, pp. 564–567] :

Geometrik araştırmada daha teknik terimleri tercih edenler

- **problem** (πρόβλημα) terimini içinde [birşey] yapılması veya inşa edilmesi önerilen [bir önerme] anlamında; ve
- **teorem** (θεώρημα) terimini içinde belirli bir hipotezin sonuçlarının ve gerekliliklerinin incelendiği [bir önerme] anlamında;

kullanırlar ama antiklerin bazıları bunların tümünü problem, bazıları da teorem olarak tarif etmiştir.

Kısaca, bir problem birşey *yapmayı* önerir; bir teorem birşeyi *görmeyi*. (Yunancada *Teorem* kelimesi daha genel olarak 'bakılmış olan' anlamındadır ve θεάομαι 'bak' fiiliyle ilgilidir; burdan ayrıca θέατρον 'theater' kelimesi de türemiştir.)

İster bir problem, ister bir teorem olsun, bir önerme—ya da daha tam anlamıyla bir önermenin *metni* —altı parçaya kadar ayrılıp analiz edilebilir. Öklid'in Heath çevirisinin *The Green Lion* baskısı [3, p. xxiii] bu analizi Proclus'un *Commentary on the First Book of Euclid's Elements* [14, p. 159] kitabında bulunan haliyle tarif eder. M.S., beşinci yüzyılda Proclus²² şöyle yazmıştır:

Bütün parçalarıyla donatılmış her problem ve teorem aşağıdaki öğeleri içermelidir:

- 1) bir **ilan** (πρότασις),

²²Proclus Bizans (yani, Konstantinapolis, şimdi İstanbul) doğumludur, ama aslında Likyalıdır, ve ilk eğitimini Ksantos'ta almıştır.

Öğelerin her önermesinin yanında, çoğu noktanın (ve bazı çizgilerin) harflerle isimlendirildiği, bir çizgi ve noktalar resmi yer alır. Bu resim **harfli diagramdır**. Her önermede diagramı kelimelerin *sonuna* yerleştiriyoruz. Reviel Netz'e göre orijinal ruloda diagram burada yer alırdı ve böylece okuyan önermeyi okumak için ruloyu ne kadar açması gerektiğini bilirdi [12, p. 35, n. 55].

Öklid'in yazdıklarının çeşitli süzgeçlerden geçmiş haline ulaşabiliyoruz. Öğelerin M. Ö. 300 civarında yazılmış olması gerekir. Bizim kullandığımız 1883'te yayınlanan Heiberg versiyonu onuncu yüzyılda Vatikan'da yazılan bir elyazmasına dayanmaktadır.

- 2) bir **açıklama** (ἐκθεσις),
- 3) bir **belirtme** (διορισμός),
- 4) bir **hazırlama** (κατασκευή),
- 5) bir **gösteri** (ἀπόδειξις), and
- 6) bir **bitirme** (συμπέρασμα).

Bunlardan, ilan, verilene ve bundan ne sonuç elde edileceğini belirtir çünkü mükemmel bir ilan bu iki parçanın ikisini de içerir. Açıklama, verilene ayrıca ele alır ve bunu daha sonra incelemede kullanılmak üzere hazırlar. Belirtme, elde edilecek sonucu ele alır ve onun ne olduğunu kesin bir şekilde açıklar. Hazırlama, elde edilecek sonuca ulaşmak için verilecek neyin eksik olduğunu söyler. Gösteri, önerilen çıkarımı kabul edilen önermelerden bilimsel akıl yürütmeye oluşturur. Bitirme, ilana geri dönerek ispatlanmış olanı onaylar.

Bir problem veya teoremin parçaları arasında en önemli olanları, her zaman bulunan, ilan, gösteri ve bitirmedir.

Biz de Proclus'un analizini aşağıdaki anlamıyla kullanacağız:

1. *İlan*, bir önermenin, harfli diagrama gönderme yapmayan, genel beyanıdır. Bu beyan, bir doğru veya üçgen gibi bir nesne hakkındadır.

2. *Açıklamada*, bu nesne diagramla harfler aracılığıyla özdeşleştirilir. Bu nesnenin varlığı üçüncü tekil emir kipinde bir fiil ile oluşturulur.

3. (a) *Belirtme*, bir *problemde*, nesne ile ilgili ne yapılacağını söyler ve δεῖ δὴ kelimeleriyle başlar. Burada δεῖ, 'gereklidir', δὴ ise 'şimdi' anlamındadır.

Felsefe öğrenmek için İskenderiye'ye ve sonra da Atina'ya gitmiştir. [14, p. xxxix].

(b) Bir *teoreme* belirtme, nesneyle ilgili neyin ispatlanacağını söyler ve ‘İddia ediyorum ki’ anlamına gelen λέγω öti kelimeleriyle başlar. Aynı ifade, bir problemde de belirtmeye ek olarak, gösterinin başında, hazırlamanın sonunda görülebilir.

4. *Hazırlamada*, eğer varsa, ikinci kelime γάρ, onaylayıcı bir zarf ve sebep belirten bir bağlaçtır. Bu kelimeyi cümlelerin birinci kelimesi ‘çünkü’ olarak çeviriyoruz.

5. *Gösteri* genellikle ἐπεὶ ‘çünkü, olduğundan’ ilgeciyle

başlar.

6. *Bitirme*, ilanı tekrarlar ve genellikle ‘dolayısıyla’ ilgecini içerir. Tekrarlanan ilandan sonra bitirme aşağıdaki iki kalıptan biriyle sonlanır:

(a) ὅπερ ἔδει ποιῆσαι ‘yapılması gereken tam buydu’ (problemlerde);

(b) ὅπερ ἔδει δεῖξαι ‘gösterilmesi gereken tam buydu’ (teoremlerde): Latince, *quod erat demonstrandum*, veya QED.

Dil

Öklid’in kullandığı dil: *Antik Yunan*cadır. Bu dil Hint-Avrupa dilleri ailesindedir. İngilizce de bu ailedendir ancak Türkçe değildir. Fakat bazı yönlerden Türkçe, Yunan-

caya, İngilizceden daha yakındır. İngilizce ve Türkçenin günümüz bilimsel terminolojisinin kökleri genellikle Yunan-

büyük	küçük	okunuş	isim
A	α	a	alfa
B	β	b	beta
Γ	γ	g	gamma
Δ	δ	d	delta
E	ε	e	epsilon
Z	ζ	z (ds)	zeta
H	η	ê (uzun e)	eta
Θ	θ	th	theta
I	ι	i	iota (yota)
K	κ	k	kappa
Λ	λ	l	lambda
M	μ	m	mü
N	ν	n	nü
Ξ	ξ	ks	ksi
O	ο	o (kisa)	omikron
Π	π	p	pi
P	ρ	r	rho (ro)
Σ	σ, ς	s	sigma
T	τ	t	tau
Υ	υ	y, ü	üpsilon
Φ	φ	f	phi
X	χ	h (kh)	khi
Ψ	ψ	ps	psi
Ω	ω	ô (uzun o)	omega

Table 7: Yunan alfabesi

Chapter 1

Elements

‘Definitions’

Boundaries ¹	Ὅροι	Sınırlar
[1] A point is [that] whose part is nothing. ²	Σημεῖόν ἐστιν, οὗ μέρος οὐθέν.	Bir nokta, parçası hiçbir şey olandır.
[2] A line, length without breadth.	Γραμμὴ δὲ μῆκος ἀπλατές.	Bir çizgi, ensiz uzunluktur.
[3] Of a line, the extremities are points.	Γραμμῆς δὲ πέρατα σημεῖα.	Bir çizginin uçlarındakiler, noktaldır.
[4] A straight line is whatever [line] evenly with the points of itself lies.	Εὐθεῖα γραμμὴ ἐστιν, ἣτις ἐξ ἴσου τοῖς ἐφ’ ἑαυτῆς σημεῖοις κεῖται.	Bir doğru, üzerindeki noktalara hizalı uzanan bir çizgidir.
[5] A surface is what has length and breadth only.	Ἐπιφάνεια δὲ ἐστιν, ὃ μῆκος καὶ πλάτος μόνον ἔχει.	Bir yüzey, sadece eni ve boyu olandır.
[6] Of a surface, the boundaries are lines.	Ἐπιφανείας δὲ πέρατα γραμμαί.	Bir yüzeyin uçlarındakiler, çizgilerdir.
[7] A plane surface is what [surface] evenly with the points of itself lies.	Ἐπίπεδος ἐπιφανεία ἐστιν, ἣτις ἐξ ἴσου ταῖς ἐφ’ ἑαυτῆς εὐθείαις κεῖται.	Bir düzlem, üzerindeki doğruların noktalarıyla hizalı uzanan bir yüzeydir.
[8] A plane angle is, ... ³ in a plane, two lines taking hold of one another, and not lying on a STRAIGHT, to one another the inclination of the lines.	Ἐπίπεδος δὲ γωνία ἐστὶν ἢ ἐν ἐπιπέδῳ δύο γραμμῶν ἀπτομένων ἀλλήλων καὶ μὴ ἐπ’ εὐθείας κειμένων πρὸς ἀλλήλας τῶν γραμμῶν κλίσις.	Bir düzlem açısı, bir düzlemde kesişen ve aynı doğru üzerinde uzan- mayan iki çizginin birbirine göre eğikliğidir.
[9] Whenever the lines containing the angle be straight, rectilinear is called the angle.	Ὅταν δὲ αἱ περιέχουσαι τὴν γωνίαν γραμμαί εὐθεῖαι ᾦσιν, εὐθύγραμμος καλεῖται ἡ γωνία.	Ve açığı içeren çizgiler biri doğru olduğu zaman düzkenar, denir açığına.
[10] Whenever a STRAIGHT,	Ὅταν δὲ εὐθεῖα	Bir doğru başka bir doğrunun üzerine yerleşip

¹The usual translation is ‘definitions’, but what follow are not really definitions in the modern sense.

²Presumably subject and predicate are inverted here, so the sense

is that of ‘A point is that of which nothing is a part.’

³There is no way to put ‘the’ here to parallel the Greek.

standing on a STRAIGHT,
the adjacent angles
equal to one another make,
right
either of the equal angles is,
and
the STRAIGHT that has been stood
is called perpendicular
to that on which it has been stood.⁴

[11] An obtuse angle is
that [which is] greater than a RIGHT.

[12] Acute,
that less than a RIGHT.

[13] A boundary is
whis is a limit of something.

[14] A figure is
what is contained by some boundary
or boundaries.⁵

[15] A circle is
a plane figure
contained by one line
[which is called the circumference]
to which,
from one point
of those lying inside of the figure
all STRAIGHTS falling
[to the circumference of the circle]
are equal to one another.

[16] A⁶ center of the circle
the point is called.

[17] A diameter of the circle is
some STRAIGHT
drawn through the center
and bounded
to either parts
by the circumference of the circle,
which also bisects the circle.

[18] A semicircle is
the figure contained
by the diameter
and the circumference taken off by it.
A center of the semicircle [is] the same
which is also of the circle.

[19] Rectilineal figures are⁷
those contained by STRAIGHTS,
triangles, by three,
quadrilaterals, by four,
polygons,⁸ by more than four
STRAIGHTS contained.

ἐπ' εὐθειᾶν σταθεῖσα
τὰς ἐφ' ἑξῆς γωνίας
ἴσας ἀλλήλαις ποιῆ,
ὀρθῇ
ἑκάτερα τῶν ἴσων γωνιῶν ἔστι,
καὶ
ἡ ἐφεστηκυῖα εὐθεῖα
κάθετος καλεῖται,
ἐφ' ἣν ἐφέστηκεν.

Ἄμβλεῖα γωνία ἔστιν
ἡ μείζων ὀρθῆς.

Ὄξεῖα δὲ
ἡ ἐλάσσων ὀρθῆς.

Ὅρος ἔστιν, ὃ τινός ἐστι πέρας.

Σχήμα ἔστι
τὸ ὑπὸ τινος ἢ τινῶν ὁρῶν πε-
ριεχόμενον.

Κύκλος ἔστι
σχῆμα ἐπίπεδον
ὑπὸ μιᾶς γραμμῆς περιεχόμενον
[ἣ καλεῖται περιφέρεια],
πρὸς ἣν
ἀφ' ἐνὸς σημείου
τῶν ἐντὸς τοῦ σχήματος κειμένων
πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι
[πρὸς τὴν τοῦ κύκλου περιφέρειαν]
ἴσαι ἀλλήλαις εἰσίν.

Κέντρον δὲ τοῦ κύκλου
τὸ σημεῖον καλεῖται.

Διάμετρος δὲ τοῦ κύκλου ἔστιν
εὐθεῖα τις
διὰ τοῦ κέντρου ἡγμένη
καὶ περατουμένη
ἐφ' ἑκάτερα τὰ μέρη
ὑπὸ τῆς τοῦ κύκλου περιφέρειας,
ἣτις καὶ δίχα τέμνει τὸν κύκλον.

Ἡμικύκλιον δὲ ἔστι
τὸ περιεχόμενον σχῆμα
ὑπὸ τε τῆς διαμέτρου
καὶ τῆς ἀπολαμβανομένης ὑπ' αὐτῆς πε-
ριφέρειας.
κέντρον δὲ τοῦ ἡμικυκλίου τὸ αὐτό,
ὃ καὶ τοῦ κύκλου ἔστιν.

Σχήματα εὐθύγραμμά ἐστι
τὰ ὑπὸ εὐθειῶν περιεχόμενα,
τρίπλευρα μὲν τὰ ὑπὸ τριῶν,
τετράπλευρα δὲ τὰ ὑπὸ τεσσάρων,
πολύπλευρα δὲ τὰ ὑπὸ πλείονων ἢ τεσ-
σάρων

birbirine eşit bitişik açılar oluşturu-
duğunda,
eşit açılarnın her birine dik açı,
ve diğerrinin üzerinde duran doğruya
da;
üzerinde durduğu doğruya bir dik
doğru denir.

Bir geniş açı,
büyük olandır bir dik açıdan.

Bir dar açı,
küçük olandır bir dik açıdan.

Bir *sınır*,
bir şeyin ucunda olandır.

Bir figür,
bir sınır tarafından veya sınırlarla
içerilendir.

Bir daire,
düzlemdeki
bir çizgiye içirilen
[bu çizgiye çember denir]
bir figürdür öyle ki
figürün içerisindeki
noktaların birinden
çizgi üzerine gelen
tüm doğrular,
birbirine eşittir;

Ve o noktaya, dairenin merkezi denir.

Bir dairenin bir çapı,
dairenin merkezinden geçip
her iki tarafta da
dairenin çevresindeki çemberce
sınırlanan
bir doğrudur
ve böyle bir doğru, daireyi ikiye böler.

Bir yarıdaire,
bir çap
ve onun kestiği bir çevrece
içirilen figürdür, ve yarıdairenin
merkezi, o dairenin merkeziyle
aynıdır.

Düzkenar figürler,
doğrularla içirilenlerdir. *Üçkenar*
figürler üç, *dörtkenar* figür-
ler dört ve *çokkenar* figürler
ise dörtten daha fazla doğruca
içirilenlerdir.

⁴This definition is quoted in Proposition 12.

⁵In Greek what is repeated is not 'boundary' but 'some'.

⁶None of the terms defined in this section is preceded by a defi-
nite article. In particular, what is being defined here is not *the* center

of a circle, but *a* center. However, it is easy to show that the center
of a given circle is unique; also, in Proposition III.1, Euclid finds *the*
center of a given circle.

[20] There being trilateral figures, an equilateral triangle is that having three sides equal, isosceles, having only two sides equal, scalene, having three unequal sides.	εὐθειῶν περιεχόμενα. ὦν δὲ τριπλευρῶν σχημάτων ἰσόπλευρον μὲν τρίγωνόν ἐστι τὸ τὰς τρεῖς ἴσας ἔχον πλευράς, ἰσοσκελὲς δὲ τὸ τὰς δύο μόνας ἴσας ἔχον πλευράς, σκαληνόν δὲ τὸ τὰς τρεῖς ἀνίσους ἔχον πλευράς.	Üçkenar figürlerden bir eşkenar üçgen, üç kenarı eşit olan, ikizkenar, eşit iki kenarı olan çeşitkenar, üç kenarı eşit olmayandır.
[21] Yet of trilateral figures, a right-angled triangle is that having a right angle, obtuse-angled, having an obtuse angle, acute-angled, having three acute angles.	Ἔτι δὲ τῶν τριπλευρῶν σχημάτων ὀρθογώνιον μὲν τρίγωνόν ἐστι τὸ ἔχον ὀρθήν γωνίαν, ἀμβλυγώνιον δὲ τὸ ἔχον ἀμβλεῖαν γωνίαν, ὀξυγώνιον δὲ τὸ τὰς τρεῖς ὀξείας ἔχον γωνίας.	Ayrıca, üçkenar figürlerden, bir dik üçgen, bir dik açısı olan, geniş açılı, bir geniş açısı olan, dar açılı, üç açısı dar açı olandır.
[22] Of quadrilateral figures, a square is what is equilateral and right-angled, an oblong, right-angled, but not equilateral, a rhombus, equilateral, but not right-angled, rhomboid, having opposite sides and angles equal, which is neither equilateral nor right-angled; and let quadrilaterals other than these be called trapezia.	Τῶν δὲ τετραπλευρῶν σχημάτων τετράγωνον μὲν ἐστίν, ὃ ἰσόπλευρόν τε ἐστὶ καὶ ὀρθογώνιον, ἑτερόμηκες δέ, ὃ ὀρθογώνιον μὲν, οὐκ ἰσόπλευρον δέ, ῥόμβος δέ, ὃ ἰσόπλευρον μὲν, οὐκ ὀρθογώνιον δέ, ῥομβοειδὲς δὲ τὸ τὰς ἀπεναντίον πλευράς τε καὶ γωνίας ἴσας ἀλλήλαις ἔχον, ὃ οὔτε ἰσόπλευρόν ἐστὶν οὔτε ὀρθογώνιον· τὰ δὲ παρὰ ταῦτα τετράπλευρα τραπέζια καλεῖσθω.	Dörtkenar figürlerden bir kare, hem eşit kenar hem de dik-açılı olan, bir dikdörtgen, dik-açılı olan ama eşit kenar olmayan, bir eşkenar dörtgen, eşit kenar olan ama dik-açılı olmayan, bir paralelkenar karşılıklı kenar ve açıları eşit olan ama eşit kenar ve dik-açılı olmayandır. Ve bunların dışında kalan dörtkenarlara yamuk denilsin.
[23] Parallels are STRAIGHTS, whichever, being in the same plane, and extended to infinity to either parts, to neither [parts] fall together with one another.	Παράλληλοι εἰσιν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὔσαι καὶ ἐχβαλλόμεναι εἰς ἄπειρον ἐφ' ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπίπτουσιν ἀλλήλαις.	Paraleller, aynı düzlemde bulunan ve her iki yönde de sınırsızca uzatıldıklarında hiçbir noktada kesişmeyen doğrulardır.

⁷As in Turkish, so in Greek, a plural subject can take a singular verb, when the subject is of the neuter gender in Greek, or names inanimate objects in Turkish.

⁸To maintain the parallelism of the Greek, we could (like Heath) use 'trilateral', 'quadrilateral', and 'multilateral' instead of 'triangle', 'quadrilateral', and 'polygon'. Today, triangles and quadrilaterals are polygons. For Euclid, they are not: you never call a triangle a polygon, because you can give the more precise information that it is a triangle.

Postulates

Postulates

Let it have been postulated
from any point
to any point
a straight line
to draw.

Also, a bounded STRAIGHT
continuously
in a straight
to extend.

Also, to any center
and distance
a circle
to draw.

Also, all right angles
equal to one another
to be.

Also, if in two straight lines
falling
the interior angles to the same parts
less than two RIGHTS make,
the two STRAIGHTS, extended
to infinity,
fall together,
to which parts are
the less than two RIGHTS.

Αιτήματα

Ἦιτήσθω
ἀπὸ παντὸς σημείου
ἐπὶ πᾶν σημεῖον
εὐθεῖαν γραμμὴν
ἀγαγεῖν.

Καὶ πεπερασμένην εὐθεῖαν
κατὰ τὸ συνεχές
ἐπ' εὐθείας
ἐκβαλεῖν.

Καὶ παντὶ κέντρῳ
καὶ διαστήματι
κύκλον
γράφεσθαι.

Καὶ πάσας τὰς ὀρθὰς γωνίας
ἴσας ἀλλήλαις
εἶναι.

Καὶ ἐὰν εἰς δύο εὐθείας εὐθεῖα
ἐμπίπτουσα
τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη γωνίας
δύο ὀρθῶν ἐλάσσονας ποιῇ,
ἐκβαλλομένας τὰς δύο εὐθείας
ἐπ' ἄπειρον
συμπίπτειν,
ἐφ' ἃ μέρη εἰσὶν
αἱ τῶν δύο ὀρθῶν ἐλάσσονες.

Postulatlar

Postulat olarak kabul edilsin
herhangi bir noktadan
herhangi bir noktaya
bir doğru
çizilmesi.

Ve sonlu bir doğrunun
kesiksiz şekilde
bir doğruya
uzatılması.

Ve her merkez
ve uzunluğa
bir daire
çizilmesi.

Ve bütün dik açılardan
bir birine eşit
olduğu.

Ve iki doğruyu
kesen bir doğrunun
aynı tarafta oluşturduğu
iç açılar iki dik açıdan küçükse,
bu iki doğrunun,
sınırsızca uzatıldıklarında
açıların
iki dik açıdan küçük olduğu tarafta
kesişeceği.

Common Notions

Common notions

Equals to the same
also to one another are equal.

Also, if to equals
equals be added,
the wholes are equal.

Also, if from equals
equals be taken away,
the remainders are equal.

Also things applying to one another
are equal to one another.

Also, the whole
than the part is greater.

Κοιναὶ ἔννοιαι

Τὰ τῶ αὐτῶ ἴσα
καὶ ἀλλήλοισ ἐστὶν ἴσα.

Καὶ ἐὰν ἴσοις
ἴσα προστεθῆ,
τὰ ὅλα ἐστὶν ἴσα.

αὶ ἐὰν ἀπὸ ἴσων
ἴσα ἀφαιρεθῆ,
τὰ καταλειπόμενά ἐστὶν ἴσα.

Καὶ τὰ ἐφαρμόζοντα ἐπ' ἀλλήλα
ἴσα ἀλλήλοισ ἐστὶν.

Καὶ τὸ ὅλον
τοῦ μέρους μείζον [ἐστὶν].

Genel Kavramlar

Aynı şeye eşitler
birbirlerine de eşittir.

Eğer eşitlere
eşitler eklenirse,
elde edilenler de eşittir.

Eğer eşitlerden
eşitler çıkartılırsa,
kalanlar eşittir.

Birbiriyle çakışan şeyler
birbirine eşittir.

Bütün,
parçadan büyüktür.

1.1

On the ¹ given bounded STRAIGHT for ² an equilateral triangle to be constructed.	Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συστήσασθαι.	Verilmiş sınırlanmış doğruya eşkenar üçgen inşa edilmesi.
Let be ³ the given bounded STRAIGHT AB.	Ἐστω ἡ δοθείσα εὐθεῖα πεπερασμένη ἡ AB.	Verilmiş sınırlanmış doğru AB olsun.
It is necessary then on the STRAIGHT AB for an equilateral triangle to be constructed. ⁴	Δεῖ δὴ ἐπὶ τῆς AB εὐθείας τρίγωνον ἰσόπλευρον συστήσασθαι.	Şimdi gereklidir AB doğrusuna eşkenar üçgenin inşa edilmesi.
To center A at distance AB suppose a circle has been drawn, [namely] BΓΔ, and moreover, to center B at distance BA suppose a circle has been drawn, [namely] AΓΕ, and from the point Γ, where the circles cut one another, to the points A and B, suppose there ⁵ have been joined the STRAIGHTS ΓA and ΓB.	Κέντρῳ μὲν τῷ A διαστήματι δὲ τῷ AB κύκλος γεγράφθω ὁ BΓΔ, καὶ πάλιν κέντρῳ μὲν τῷ B διαστήματι δὲ τῷ BA κύκλος γεγράφθω ὁ AΓΕ, καὶ ἀπὸ τοῦ Γ σημείου, καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι, ἐπὶ τὰ A, B σημεία ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓA, ΓB.	A merkezine, AB uzaklığında olan çember çizilmiş olsun, BΓΔ, ve yine B merkezine, BA uzaklığında olan çember çizilmiş olsun, AΓΕ, çemberlerin kesiştiği Γ noktasından A, B noktalarına ΓA, ΓB doğruları birleştirilmiş olsun.
And since the point A is the center of the circle ΓΔB, equal is AΓ to AB; moreover, since the point B is the center of the circle ΓAΕ, equal is BΓ to BA.	Καὶ ἐπεὶ τὸ A σημεῖον κέντρον ἐστὶ τοῦ ΓΔB κύκλου, ἴση ἐστὶν ἡ AΓ τῇ AB· πάλιν, ἐπεὶ τὸ B σημεῖον κέντρον ἐστὶ τοῦ ΓAΕ κύκλου, ἴση ἐστὶν ἡ BΓ τῇ BA.	Ve A noktası ΓΔB çemberinin merkezi olduğu için, AΓ, AB doğrusuna eşittir. Dahası B noktası ΓAΕ çemberinin merkezi olduğu için, BΓ, BA doğrusuna eşittir.

¹Heath's translation has the indefinite article 'a' here, in accordance with modern mathematical practice. However, Euclid does use the Greek *definite* article here, just as in the *exposition* (see §). In particular, he uses the definite article as a *generic* article, which 'makes a single object the representative of the entire class' [16, ¶1123, p. 288]. English too has a generic use of the definite article, 'to indicate the class or kind of objects, as in the well-known aphorism: *The child is the father of the man*' [6, p. 76]. (However, the enormous *Cambridge Grammar* does not discuss the generic article in the obvious place [7, 5.6.1, pp. 568–71]. By the way, the 'well-known aphorism' is by Wordsworth; see http://en.wikisource.org/wiki/0de:_Intimations_of_Immortality_from_Recollections_of_Early_Childhood [accessed July 27, 2011].) See note 1 to Proposition 9 below.

²The Greek form of the enunciation here is an infinitive clause, and the subject of such a clause is generally in the accusative case [16, ¶1972, p. 438]. In English, an infinitive clause with expressed subject (as here) is always preceded by 'for' [7, 14.1.3, p. 1178]. Normally such a clause, in Greek or English, does not stand by itself as a complete sentence; here evidently it is expected to. Note that the Greek infinitive is thought to be originally a noun in the dative case [16, ¶1969, p. 438]; the English infinitive with 'to' would seem to be formed similarly.

³We follow Euclid in putting the verb (a third-person imperative) first; but a smoother translation of the exposition here would be, 'Let the given finite straight line be AB.' Heath's version is, 'Let AB be the given finite straight line.' By the argument of Netz [12, pp. 43–4], this would appear to be a misleading translation, if not a mistranslation. Euclid's expression ἡ AB, 'the AB', must be understood as an abbreviation of ἡ εὐθεῖα γραμμὴ ἡ AB or ἡ AB εὐθεῖα γραμμή, 'the

straight line AB'. In Proposition XIII.4, Euclid says, Ἐστω εὐθεῖα ἡ AB, which Heath translates as 'Let AB be a straight line'; but then this suggests the expansion 'Let the straight line AB be a straight line', which does not make much sense. Netz's translation is, 'Let there be a straight line, [namely] AB.' The argument is that Euclid does *not* use words to establish a correlation between letters like A and B and points. The correlation has already been established in the diagram that is before us. By saying, Ἐστω εὐθεῖα ἡ AB, Euclid is simply calling our attention to a part of the diagram. Now, in the present proposition, Heath's translation of the exposition is expanded to, 'Let the straight line AB be the given finite straight line', which does seem to make sense, at least if it can be expanded further to 'Let the finite straight line AB be the given finite straight line.' But, unlike AB, the given finite straight line was already mentioned in the enunciation, so it is less misleading to name this first in the exposition.

⁴Slightly less literally, 'It is necessary that on the STRAIGHT AB, an equilateral triangle be constructed.'

⁵Instead of 'suppose there have been joined', we could write 'let there have been joined'. However, each of these translations of a Greek *third-person imperative* begins with a second-person imperative (because there is no third-person imperative form in English, except in some fixed forms like 'God bless you'). The logical subject of the verb 'have been joined' is 'the STRAIGHT AB'; since this comes after the verb, it would appear to be an *extraposed subject* in the sense of the *Cambridge Grammar of the English Language* [7, 2.16, p. 67]. Then the grammatical subject of 'have been joined' is 'there', used as a *dummy*; but it will not always be appropriate to use a dummy in such situations [7, 16.63, p. 1402–3].

And ΓA was shown equal to AB ;
therefore either of ΓA and ΓB to AB
is equal.
But equals to the same
are also equal to one another;
therefore also ΓA is equal to ΓB .
Therefore the three ΓA , AB , and $B\Gamma$
are equal to one another.

Equilateral therefore
is triangle $AB\Gamma$.

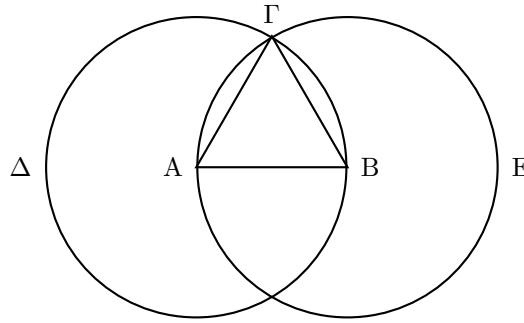
Also, it has been constructed
on the given bounded STRAIGHT
 AB ;
—just what it was necessary to do.

ἔδειχθη δὲ καὶ ἡ ΓA τῆ AB ἴση·
ἕκατέρα ἄρα τῶν ΓA , ΓB τῆ AB
ἴσταν ἴση.
τὰ δὲ τῶ αὐτῶ ἴσα
καὶ ἀλλήλοις ἴσταν ἴσα·
καὶ ἡ ΓA ἄρα τῆ ΓB ἴσταν ἴση·
αἱ τρεῖς ἄρα αἱ ΓA , AB , $B\Gamma$
ἴσαι ἀλλήλαις εἰσίν.

Ἰσόπλευρον ἄρα
ἔσταν τὸ $AB\Gamma$ τρίγωνον.
καὶ συνέσταται
ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης
τῆς AB .⁶
ὅπερ ἔδει ποιῆσαι.

Ve ΓA doğrusunun, AB doğrusuna eşit
olduğu gösterilmişti.
O zaman ΓA , ΓB doğrularının her biri
 AB doğrusuna eşittir.
Ama aynı şeye eşit olanlar
birbirine eşittir.
O zaman ΓA , ΓB doğrusuna eşittir.
O zaman o üç doğru, ΓA , AB , $B\Gamma$,
birbirine eşittir.

Eşkenardır dolayısıyla,
 $AB\Gamma$ üçgeni
ve inşa edilmiştir
verilmiş sınırlanmış,
 AB doğrusuna;
—yapılması gereken tam buydu.



1.2

At the given point,
equal to the given STRAIGHT,
for a STRAIGHT to be placed.

Let be
the given point A ,
and the given STRAIGHT, $B\Gamma$.

It is necessary then
at the point A
equal to the given STRAIGHT $B\Gamma$
for a STRAIGHT to be placed.

For, suppose there has been joined
from the point A to the point B
a STRAIGHT, AB ,
and there has been constructed on it
an equilateral triangle, ΔAB ,
and suppose there have been extended
on a STRAIGHT¹ with ΔA and ΔB
the STRAIGHTS AE and BZ ,
and to the center B
at distance $B\Gamma$
suppose a circle has been drawn,
 $\Gamma H\Theta$,
and again to the center Δ
at distance ΔH
suppose a circle has been drawn,

Πρὸς τῶ δοθέντι σημείῳ
τῆ δοθείση εὐθεία ἴσην
εὐθεῖαν θέσθαι.

Ἐσταν
τὸ μὲν δοθὲν σημεῖον τὸ A ,
ἡ δὲ δοθείσα εὐθεῖα ἡ $B\Gamma$.

δεῖ δὴ
πρὸς τῶ A σημείῳ
τῆ δοθείση εὐθεία τῆ $B\Gamma$ ἴσην
εὐθεῖαν θέσθαι.

Ἐπεζεύχθω γὰρ
ἀπὸ τοῦ A σημείου ἐπὶ τὸ B σημεῖον
εὐθεῖα ἡ AB ,
καὶ συνεστάτω ἐπ' αὐτῆς
τρίγωνον ἰσόπλευρον τὸ ΔAB ,
καὶ ἐκβεβλήσθωσαν
ἐπ' εὐθείας ταῖς ΔA , ΔB
εὐθεῖαι αἱ AE , BZ ,
καὶ κέντρῳ μὲν τῶ B
διαστήματι δὲ τῶ $B\Gamma$
κύκλος γεγράφθω
ὁ $\Gamma H\Theta$,
καὶ πάλιν κέντρῳ τῶ Δ
καὶ διαστήματι τῶ ΔH
κύκλος γεγράφθω

Verilmiş noktaya
verilmiş doğruya eşit olan
bir doğrunun konulması.

Verilmiş nokta A olsun,
verilmiş doğru $B\Gamma$.

Gereklidir
 A noktasına,
 $B\Gamma$ doğrusuna eşit olan
bir doğrunun konulması.

Çünkü, birleştirilmiş olsun
 A noktasından B noktasına,
 AB doğrusu,
ve bu doğru üzerine inşa edilmiş olsun
eşkenar üçgen ΔAB ,
ve uzatılmış olsun,
 ΔA , ΔB doğrularından
 AE , BZ doğruları
ve B merkezine,
 $B\Gamma$ uzaklığında,
çizilmiş olsun,
 $\Gamma H\Theta$ çemberi ve yine Δ merkezine,
 ΔH uzaklığında
çizilmiş olsun,
 HKA çemberi .

⁶Normally Heiberg puts a semicolon at this position. Perhaps he has a period here only because he has bracketed the following words (omitted here): 'Therefore, on a given bounded STRAIGHT,

an equilateral triangle has been constructed.' According to Heiberg, these words are found, not in the manuscripts of Euclid, but in Proclus's commentary [14, p. 210] alone.

HKΛ.

Since then the point B is the center of ΓHΘ,

BΓ is equal to BH.

Moreover,

since the point Δ is the center of the circle KHA,

equal is ΔA to ΔH;

of these, the [part] ΔA to ΔB is equal.

Therefore the remainder AA to the remainder BH is equal.

But BΓ was shown equal to BH.

Therefore either of AA and BΓ to BH is equal.

But equals to the same

also are equal to one another.

And therefore AA is equal to BΓ.

Therefore at the given point A equal to the given STRAIGHT BΓ the STRAIGHT AA is laid down; —just what it was necessary to do.

ὁ HKΛ.

Ἐπεὶ οὖν τὸ B σημεῖον κέντρον ἐστὶ τοῦ ΓHΘ,

ἴση ἐστὶν ἡ BΓ τῆς BH.

πάλιν,

ἐπεὶ τὸ Δ σημεῖον κέντρον ἐστὶ τοῦ HKΛ κύκλου,

ἴση ἐστὶν ἡ ΔA τῆς ΔH,

ὣν ἡ ΔA τῆς ΔB

ἴση ἐστὶν.

λοιπῆ ἄρα ἡ AA

λοιπῆ τῆς BH

ἐστὶν ἴση.

ἐδείχθη δὲ καὶ ἡ BΓ τῆς BH ἴση·

ἐκατέρω ἄρα τῶν AA, BΓ τῆς BH

ἐστὶν ἴση.

τὰ δὲ τῶ αὐτῶ ἴσα

καὶ ἀλλήλοις ἐστὶν ἴσα·

καὶ ἡ AA ἄρα τῆς BΓ ἐστὶν ἴση.

Πρὸς ἄρα τῶ δοθέντι σημείῳ τῶ A τῆς δοθείσης εὐθείας τῆς BΓ ἴση εὐθεῖα κείται ἡ AA· ὅπερ ἔδει ποιῆσαι.

B noktası ΓHΘ çemberinin merkezi olduđu için,

BΓ, BH doğrusuna eşittir.

Yine,

Δ noktası HKΛ çemberinin merkezi olduđu için,

ΔA, ΔH doğrusuna eşittir,

ve (birincinin) ΔA parçası,

(ikincinin) ΔB parçasına eşittir.

Dolayısıyla AA kalanı,

BH kalanına

eşittir.

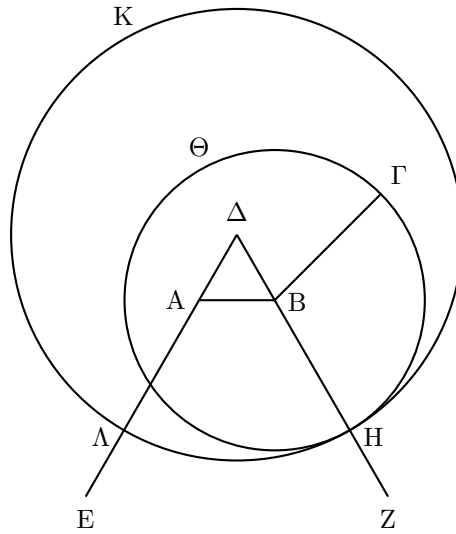
Ve BΓ doğrusunun, BH doğrusuna eşit olduđu gösterilmişti.

Dolayısıyla AA, BΓ doğrularının her biri BH doğrusuna eşittir.

Ama aynı şeye eşit olanlar birbirine eşittir.

Ve dolayısıyla AA da, BΓ doğrusuna eşittir.

Dolayısıyla verilmiş A noktasına verilmiş BΓ doğrusuna eşit olan AA doğrusu konulmuştur; —yapılması gereken tam buydu.



1.3

Two unequal STRAIGHTS being given, from the greater, equal to the less, a STRAIGHT to take away.

Let be the two given unequal STRAIGHTS AB and Γ,¹ of which let the greater be AB.

It is necessary then from the greater, AB,

Δύο δοθεισῶν εὐθειῶν ἀνίσων ἀπὸ τῆς μείζονος τῆς ἐλάσσονος ἴσην εὐθεῖαν ἀφελεῖν.

Ἐστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι ἀνισοὶ αἱ AB, Γ, ὣν μείζων ἔστω ἡ AB·

δεῖ δὴ ἀπὸ τῆς μείζονος τῆς AB

İki eşit olmayan doğru verilmiş ise, daha büyükten daha küçüğe eşit olan bir doğru kesmek.

İki verilmiş doğru AB, Γ olsunlar; daha büyüğü AB olsun.

Gereklidir daha büyük olan AB doğrusundan

¹The phrase ἐπ' εὐθείας will recur a number of times. The adjective, which is feminine here, appears to be a genitive singular, though it could be accusative plural.

¹Since Γ is given the feminine gender in the Greek, this is a sign that Γ is indeed a line and not a point. See the Introduction.

equal to the less, Γ ,
to take away a STRAIGHT.

Let there be laid down
at the point A,
equal to the line Γ ,
 $A\Delta$;
and to center A
at distance $A\Delta$
suppose circle ΔEZ has been drawn.

And since the point A
is the center of the circle ΔEZ ,
equal is AE to $A\Delta$.
But Γ to $A\Delta$ is equal.
Therefore either of AE and Γ
is equal to $A\Delta$;
and so AE is equal to Γ .

Therefore, two unequal STRAIGHTS
being given, AB and Γ ,
from the greater, AB,
an equal to the less, Γ ,
has been taken away, [namely] AE;
—just what it was necessary to do.

τῆ ἐλάσσονι τῆ Γ ἴσην
εὐθεῖαν ἀφελεῖν.

Κεῖσθω
πρὸς τῷ A σημείῳ
τῆ Γ εὐθείᾳ ἴση
ἡ $A\Delta$.
καὶ κέντρῳ μὲν τῷ A
διαστήματι δὲ τῷ $A\Delta$
κύκλος γεγράφθω ὁ ΔEZ .

Καὶ ἐπεὶ τὸ A σημεῖον
κέντρον ἐστὶ τοῦ ΔEZ κύκλου,
ἴση ἐστὶν ἡ AE τῆ $A\Delta$.
ἀλλὰ καὶ ἡ Γ τῆ $A\Delta$ ἐστὶν ἴση.
ἐκατέρα ἄρα τῶν AE, Γ
τῆ $A\Delta$ ἐστὶν ἴση.
ὥστε καὶ ἡ AE τῆ Γ ἐστὶν ἴση.

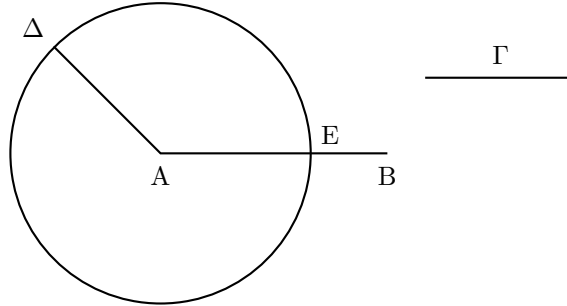
Δύο ἄρα δοθεισῶν εὐθειῶν ἀνίσων τῶν
AB, Γ
ἀπὸ τῆς μείζονος τῆς AB
τῆ ἐλάσσονι τῆ Γ ἴση
ἀφῆρηται ἡ AE.
ὅπερ ἔδει ποιῆσαι.

daha küçük olan Γ doğrusuna eşit olan
bir doğru kesmek.

Konulsun
A noktasına
 Γ doğrusuna eşit olan
 $A\Delta$ doğrusu.
Ve A merkezine
 $A\Delta$ uzaklığında olan
 ΔEZ çemberi çizilmiş olsun.

Ve A noktası
 ΔEZ çemberinin merkezi olduğu için,
AE, $A\Delta$ doğrusuna eşittir.
Ama Γ , $A\Delta$ doğrusuna eşittir.
Dolayısıyla AE, Γ doğrularının her
biri
 $A\Delta$ doğrusuna eşittir.
Sonuç olarak,
AE, Γ doğrusuna eşittir.

Dolayısıyla iki eşit olmayan AB, Γ
doğrusu verilmiş ise,
daha büyük olan AB doğrusundan
daha küçük olan Γ doğrusuna eşit olan
AE doğrusu kesilmişti;
—yapılması gereken tam buydu.



1.4

If two triangles
two sides
to two sides
have equal,¹
either [side] to either,²
and angle to angle have equal,
—that which is by the equal
STRAIGHTS³
contained,
also⁴ base to base
they will have equal,
and the triangle to the triangle
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δυοῖ πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα
καὶ τὴν γωνίαν τῆ γωνία ἴσην ἔχη
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν τῆ βάσει
ἴσην ἔξει,
καὶ τὸ τρίγωνον τῷ τριγώνῳ
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἐκατέρα ἐκατέρα,
ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν.

Eğer iki üçgende
iki kenar
iki kenara
eşit olursa
(her biri birine)
ve açı açığa eşit olursa
(yani, eşit doğrular tarafından
içerilen),
hem taban tabana
eşit olacak,
hem üçgen üçgene
eşit olacak,
hem de geriye kalan açılar
geriye kalan açılara
eşit olacak,
her biri birine,
(yani) eşit kenarları görenler.

—those that the equal sides subtend.

Let be
two triangles $AB\Gamma$ and ΔEZ ,
the two sides AB and AG
to the two sides ΔE and ΔZ
having equal,
either to either,
 AB to ΔE and AG to ΔZ ,
and angle BAG
to $E\Delta Z$
equal.

I say that
the base $B\Gamma$ is equal to the base EZ ,
and triangle $AB\Gamma$
will be equal to triangle ΔEZ ,
and the remaining angles
to the remaining angles
will be equal,
either to either,
those that equal sides subtend,
[namely] $AB\Gamma$ to ΔEZ ,
and AGB to ΔZE .

For, there being applied
triangle $AB\Gamma$
to triangle ΔEZ ,
and there being placed
the point A on the point Δ ,
and the STRAIGHT AB on ΔE ,
also the point B will apply⁵ to E ,
by the equality of AB to ΔE .
Then, AB applying to ΔE ,
also STRAIGHT AG will apply to ΔZ ,
by the equality
of angle BAG to $E\Delta Z$.
Hence the point Γ to the point Z
will apply,
by the equality, again, of AG to ΔZ .
But B had applied to E ;
Hence the base $B\Gamma$ to the base EZ
will apply.
For if,
 B applying to E ,
and Γ to Z ,
the base $B\Gamma$ will not apply to EZ ,
two STRAIGHTS will enclose a space,
which is impossible.
Therefore will apply
base $B\Gamma$ to EZ
and will be equal to it.
Hence triangle $AB\Gamma$ as a whole

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , AG
ταῖς δυοὶ πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἑκατέραν ἑκατέρῃ
τὴν μὲν AB τῇ ΔE τὴν δὲ AG τῇ ΔZ
καὶ γωνίαν τὴν ὑπὸ BAG
γωνία τῇ ὑπὸ $E\Delta Z$
ἴσην.

λέγω, ὅτι
καὶ βάσις ἡ $B\Gamma$ βάσει τῇ EZ ἴση ἐστίν,
καὶ τὸ $AB\Gamma$ τρίγωνον
τῷ ΔEZ τριγώνῳ ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἑκατέρα ἑκατέρῃ,
ὑφ' ἃς αἱ ἴσαι πλευραὶ ὑποτείνουσιν,
ἡ μὲν ὑπὸ $AB\Gamma$ τῇ ὑπὸ ΔEZ ,
ἡ δὲ ὑπὸ AGB τῇ ὑπὸ ΔZE .

Ἐφαρμοζομένου γὰρ
τοῦ $AB\Gamma$ τριγώνου
ἐπὶ τὸ ΔEZ τρίγωνον
καὶ τιθεμένου
τοῦ μὲν A σημείου ἐπὶ τὸ Δ σημεῖον
τῆς δὲ AB εὐθείας ἐπὶ τὴν ΔE ,
ἐφαρμόσει καὶ τὸ B σημεῖον ἐπὶ τὸ E
διὰ τὸ ἴσην εἶναι τὴν AB τῇ ΔE .
ἐφαρμοσάσης δὲ τῆς AB ἐπὶ τὴν ΔE
ἐφαρμόσει καὶ ἡ AG εὐθεῖα ἐπὶ τὴν ΔZ
διὰ τὸ ἴσην εἶναι
τὴν ὑπὸ BAG γωνίαν τῇ ὑπὸ $E\Delta Z$.
ὥστε καὶ τὸ Γ σημεῖον ἐπὶ τὸ Z σημεῖον
ἐφαρμόσει
διὰ τὸ ἴσην πάλιν εἶναι τὴν AG τῇ ΔZ .
ἀλλὰ μὴν καὶ τὸ B ἐπὶ τὸ E ἐφαρμόκει·
ὥστε βάσις ἡ $B\Gamma$ ἐπὶ βάσιν τὴν EZ
ἐφαρμόσει.
εἰ γὰρ
τοῦ μὲν B ἐπὶ τὸ E ἐφαρμόσαντος
τοῦ δὲ Γ ἐπὶ τὸ Z
ἡ $B\Gamma$ βάσις ἐπὶ τὴν EZ οὐκ ἐφαρμόσει,
δύο εὐθεῖαι χωρίον περιέξουσιν·
ὅπερ ἐστὶν ἀδύνατον.
ἐφαρμόσει ἄρα
ἡ $B\Gamma$ βάσις ἐπὶ τὴν EZ
καὶ ἴση αὐτῇ ἔσται·
ὥστε καὶ ὅλον τὸ $AB\Gamma$ τρίγωνον

Verilmiş olsun,
 $AB\Gamma$ ve ΔEZ (adlarında) iki üçgen,
iki kenarı AB , AG
 ΔE , ΔZ iki kenarıma
eşit olan
her biri birine,
(şöyle ki) AB , ΔE kenarıma ve AG , ΔZ
kenarıma,
ve BAG (tarafından içerilen) açısı
 $E\Delta Z$ açısına
eşit olan.

İddia ediyorum ki,
 $B\Gamma$ tabanı eşittir EZ tabanıma,
ve $AB\Gamma$ üçgeni
eşit olacak ΔEZ üçgenine,
ve geriye kalan açılar eşit olacak geriye
kalan açılara,
her biri birine,
(şöyle ki) eşit kenarları görenler;
 $AB\Gamma$, ΔEZ açısına,
 AGB , ΔZE açısına.

Çünkü, üstüne koyulursa
 $AB\Gamma$ üçgeni
 ΔEZ üçgeninin,
ve yerleştirilirse
 A noktası Δ noktasma,
ve AB doğrusu ΔE doğrusuna,
o zaman B noktası yerleşecek E nok-
tasına,
 AB doğrusunun ΔE doğrusuna eşitliği
sayesinde.
Böylece, AB doğrusunu yerleştirilince
 ΔE doğrusuna,
 AG doğrusu üstüne gelecek ΔZ
doğrusunun,
 BAG açısının eşitliği sayesinde,
 $E\Delta Z$ açısına.
Dolayısıyla, Γ noktası yerleşecek Z
noktasına,
eşitliği sayesinde, yine, AG doğrusu-
nun ΔZ doğrusuna.
Ama B konuldu E noktasma;
Dolayısıyla, $B\Gamma$ tabanı üstüne gelecek
 EZ tabanıma.
Çünkü eğer, konulunca B , E nok-
tasına,
ve Γ , Z noktasma,
 $B\Gamma$ tabanı yerleşmeyecekse EZ ta-
banına,

¹More smoothly, 'If two triangles have two sides equal to two sides'.

²That is, 'respectively'. We could translate the Greek also as 'each to each'; but the Greek *ἑκατέρος* has the dual number, as opposed to *ἕκαστος* 'each'. The English form 'either' is a remnant of the dual number.

³It appears that for Euclid, things are never simply *equal*; they are equal *to* something. Here the equal STRAIGHTS containing the angle are not equal to one another; they are separately equal to the two STRAIGHTS in the other triangle.

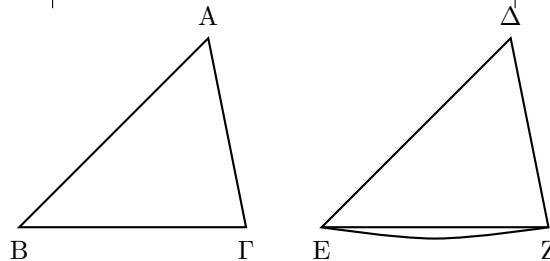
⁴Here Euclid's *καί* has a different meaning from the earlier instance; now it shows the transition to the conclusion of the enunciation. In fact the conclusion has the form *καί...καί...καί...*. This general form might be translated as 'Both...and...and...' The word *both* properly refers to two things, but the Oxford English Dictionary cites an example from Chaucer (1386) where it refers to three things: 'Both heaven and earth and sea'. The word *both* seems to have entered English late, from Old Norse; it supplanted the earlier *wordbo*.

to triangle $\triangle EZ$ as a whole
will apply
and will be equal to it,
and the remaining angles
to the remaining angles
will apply,
and be equal to them,
 $AB\Gamma$ to $\triangle EZ$
and AGB to $\triangle ZE$.

If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
and angle to angle have equal,
—that which is by the equal
STRAIGHTS
contained,
also base to base
they will have equal,
and the triangle to the triangle
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,
—those that the equal sides subtend;
—just what it was necessary to show.

ἐπὶ ὅλον τὸ $\triangle EZ$ τριγώνων
ἐφαρμοσέει
καὶ ἴσον αὐτῷ ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ἐπὶ τὰς λοιπὰς γωνίας
ἐφαρμοσούσι
καὶ ἴσαι αὐταῖς ἔσονται,
ἢ μὲν ὑπὸ $AB\Gamma$ τῇ ὑπὸ $\triangle EZ$
ἢ δὲ ὑπὸ AGB τῇ ὑπὸ $\triangle ZE$.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχη
ἑκατέραν ἑκατέρᾳ
καὶ τὴν γωνίαν τῇ γωνίᾳ ἴσην ἔχη
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν τῇ βάσει
ἴσην ἔξει,
καὶ τὸ τρίγωνον τῷ τριγώνῳ
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἑκατέρα ἑκατέρᾳ,
ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν·
ὅπερ ἔδει δεῖξαι.



iki doğru çevreleyecek bir alan,
imkansız olan.
Bu yüzden $B\Gamma$ tabanı çakışacak EZ
tabanıyla
ve eşit olacak ona.
Dolayısıyla $AB\Gamma$ üçgeninin tamamı
üstüne gelecek $\triangle EZ$ üçgeninin
tamamına,
ve eşit olacak ona,
ve geriye kalan açılar üstüne gelecekler
geriye kalan açılarını,
ve eşit olacaklar onlara;
 $AB\Gamma$, $\triangle EZ$ açısına
ve AGB , $\triangle ZE$ açısına.

Dolayısıyla, eğer,
iki üçgenin, varsa iki kenarı eşit olan
iki kenara,
her bir (kenar) birine,
ve varsa açığı eşit açısı,
(yani) eşit doğrularca içerilen,
hem tabana eşit tabanları olacak,
hem üçgen eşit olacak üçgene,
hem de geriye kalan açılar eşit olacak
geriye kalan açılarını,
her biri birine,
(yani) eşit kenarları görenler;
—gösterilmesi gereken tam buydu.

1.5

In¹ isosceles triangles,
the angles at the base
are equal to one another,
and,
the equal STRAIGHTS being extended,
the angles under the base
will be equal to one another.

Let there be
an isosceles triangle, $AB\Gamma$
having equal
side AB to side AG ,
and suppose have been extended
on a STRAIGHT with AB and AG

Τῶν ἰσοσκελῶν τριγώνων
αἱ πρὸς τῇ βάσει γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ
προσεκβληθεισῶν τῶν ἴσων εὐθειῶν
αἱ ὑπὸ τὴν βάσιν γωνίαι
ἴσαι ἀλλήλαις ἔσονται.

Ἐστω
τρίγωνον ἰσοσκελὲς τὸ $AB\Gamma$
ἴσην ἔχον
τὴν AB πλευρὰν τῇ AG πλευρᾷ,
καὶ προσεκβεβλήσθωσαν
ἐπ' εὐθείας ταῖς AB , AG

İkizkenar üçgenlerde,
tabandaki açılar,
birbirine eşittir,
ve,
eşit doğrular uzatıldığında,
tabanın altında kalan açılar,
birbirine eşit olacaklar.

Verilmiş olsun,
bir $AB\Gamma$ ikizkenar üçgeni;
 AB kenarı eşit olan AG kenarına,
ve varsayılın $B\Delta$ ve ΓE doğrularının
uzatılmış olduğu, AB ve AG
doğrularından.

¹Heath has *coinciding* here, but the verb is just the active form of what, in the passive, is translated as *being applied*.

¹More literally, 'of'.

the STRAIGHTS $B\Delta$ and ΓE .

I say that
angle $AB\Gamma$ to angle $A\Gamma B$
is equal,
and $\Gamma B\Delta$ to $B\Gamma E$.

For, suppose there has been chosen
a random point Z on $B\Delta$,
and there has been taken away
from the greater, AE ,
to the less, AZ ,
an equal, AH ,
and suppose there have been joined
the STRAIGHTS $Z\Gamma$ and HB .

Since then AZ is equal to AH ,
and AB to $A\Gamma$,
so the two AZ and $A\Gamma$
to the two HA , AB ,
will be equal,
either to either;
and they bound a common angle,
[namely] ZAH ;
therefore the base $Z\Gamma$ to the base HB
is equal,
and triangle $AZ\Gamma$ to triangle AHB
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,
those that the equal sides subtend,
 $A\Gamma Z$ to ABH ,
and $AZ\Gamma$ to AHB .
And since AZ as a whole
to AH as a whole
is equal,
of which the [part] AB to $A\Gamma$ is equal,
therefore the remainder BZ
to the remainder ΓH
is equal.
And $Z\Gamma$ was shown equal to HB .
Then the two BZ and $Z\Gamma$
to the two ΓH and HB
are equal,
either to either,
and angle $BZ\Gamma$
to angle ΓHB
[is] equal,
and the common base of them is $B\Gamma$;
and therefore triangle $BZ\Gamma$
to triangle ΓHB
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,
which the equal sides subtend.
Equal therefore is
 $ZB\Gamma$ to $H\Gamma B$,
and $B\Gamma Z$ to ΓBH .
Since then angle ABH as a whole

εὐθεΐαι αἱ $B\Delta$, ΓE .

λέγω, ὅτι
ἡ μὲν ὑπὸ $AB\Gamma$ γωνία τῆ ὑπὸ $A\Gamma B$
ἴση ἐστίν,
ἡ δὲ ὑπὸ $\Gamma B\Delta$ τῆ ὑπὸ $B\Gamma E$.

Εἰλήφθω γὰρ
ἐπὶ τῆς $B\Delta$ τυχὸν σημεῖον τὸ Z ,
καὶ ἀφῆρήσθω
ἀπὸ τῆς μείζονος τῆς AE
τῆ ἐλάσσονι τῆ AZ
ἴση ἡ AH ,
καὶ ἐπεζεύχθωσαν
αἱ $Z\Gamma$, HB εὐθεΐαι.

Ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν AZ τῆ AH
ἡ δὲ AB τῆ $A\Gamma$,
δύο δὴ αἱ ZA , $A\Gamma$
δυσὶ ταῖς HA , AB
ἴσαι εἰσὶν
ἐκατέρω ἐκατέρω·
καὶ γωνίαν κοινὴν περιέχουσι
τὴν ὑπὸ ZAH .
βάσις ἄρα ἡ $Z\Gamma$ βάσει τῆ HB
ἴση ἐστίν,
καὶ τὸ $AZ\Gamma$ τρίγωνον τῷ AHB τριγώνω
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωναὶ
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἐκατέρω ἐκατέρω,
ὅφ' ἄς αἱ ἴσαι πλευραὶ ὑποτείνουσιν,
ἡ μὲν ὑπὸ $A\Gamma Z$ τῆ ὑπὸ ABH ,
ἡ δὲ ὑπὸ $AZ\Gamma$ τῆ ὑπὸ AHB .
καὶ ἐπεὶ ὅλη ἡ AZ
ὅλη τῆ AH
ἐστὶν ἴση,
ὣν ἡ AB τῆ $A\Gamma$ ἐστὶν ἴση,
λοιπὴ ἄρα ἡ BZ
λοιπῆ τῆ ΓH
ἐστὶν ἴση.
ἐδείχθη δὲ καὶ ἡ $Z\Gamma$ τῆ HB ἴση.
δύο δὴ αἱ BZ , $Z\Gamma$
δυσὶ ταῖς ΓH , HB
ἴσαι εἰσὶν
ἐκατέρω ἐκατέρω·
καὶ γωνία ἡ ὑπὸ $BZ\Gamma$
γωνία τῆ ὑπὸ ΓHB
ἴση,
καὶ βάσις αὐτῶν κοινὴ ἡ $B\Gamma$.
καὶ τὸ $BZ\Gamma$ ἄρα τρίγωνον
τῷ ΓHB τριγώνω
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωναὶ
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἐκατέρω ἐκατέρω,
ὅφ' ἄς αἱ ἴσαι πλευραὶ ὑποτείνουσιν·
ἴση ἄρα ἐστὶν
ἡ μὲν ὑπὸ $ZB\Gamma$ τῆ ὑπὸ $H\Gamma B$
ἡ δὲ ὑπὸ $B\Gamma Z$ τῆ ὑπὸ ΓBH .
ἐπεὶ οὖν ὅλη ἡ ὑπὸ ABH γωνία

İddia ediyorum ki
 $AB\Gamma$ açısı, $A\Gamma B$ açısına,
eşittir
ve $\Gamma B\Delta$ açısı eşittir $B\Gamma E$ açısına.

Çünkü, kabul edelim ki, seçilmiş ol-
sun,
rastgele bir Z noktası $B\Delta$ üzerinde,
ve AH ,
büyük olan AE doğrusundan
küçük olan AZ doğrusunun kesilmiş
olsun,
ve $Z\Gamma$ ile HB birleştirilmiş olsun.

Çünkü o zaman AZ eşittir AH
doğrusuna,
ve AB doğrusu $A\Gamma$ doğrusuna,
böylece AZ ve $A\Gamma$ ikilisi eşit olacak
 HA ve AB ikilisinin,
her biri birine;
ve sınırlandırılır ortak bir açıyı,
(yani) ZAH açısını;
dolayısıyla $Z\Gamma$ tabanı eşittir HB ta-
banına,
ve $AZ\Gamma$ üçgeni eşit olacak AHB üçge-
nine,
ve geriye kalan açılar eşit olacaklar
geriye kalan açılarının,
her biri birine,
(yani) eşit kenarları görenler;
 $A\Gamma Z$ açısı ABH açısına,
ve $AZ\Gamma$ açısı AHB açısına.
Böylece AZ bütününe eşitliği AH
bütününe,
ve bunların AB parçasının eşitliği $A\Gamma$
parçasına,
gerekirir BZ kalanının eşit olmasını
 ΓH kalanına.
Ve $Z\Gamma$ doğrusunun gösterilmişti eşit
olduğu HB doğrusuna.
O zaman BZ ve $Z\Gamma$ ikilisi eşittir ΓH ve
 HB ikilisinin,
her biri birine,
ve $BZ\Gamma$ açısı ΓHB açısına,
ve onların ortak tabanı $B\Gamma$
doğrusudur;
ve bu yüzden $BZ\Gamma$ üçgeni eşit olacak
 ΓHB üçgenine,
ve geriye kalan açılar da eşit olacaklar
geriye kalan açılarının,
her biri birine,
aynı kenarları görenler.
Dolayısıyla $ZB\Gamma$ eşittir $H\Gamma B$ açısına,
ve $B\Gamma Z$ açısı ΓBH açısına.
Çünkü gösterilmiş oldu ABH açısının
bütününe eşit olduğu $A\Gamma Z$
açısının bütününe,
ve bunların ΓBH parçasının (eşitliği)
 $B\Gamma Z$ parçasına,
dolayısıyla $AB\Gamma$ kalanı eşittir $A\Gamma B$
kalanına;

to angle AGZ as a whole
was shown equal,
of which the [part] GBH to BGZ
is equal,
therefore the remainder ABF
to the remainder AGB
is equal;
and they are at the base
of the triangle ABG .
And was shown also
 ZBF equal to HGB ;
and they are under the base.

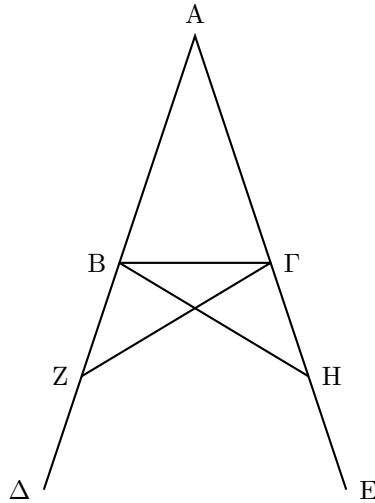
Therefore, in isosceles triangles,
the angles at the base
are equal to one another,
and,
the equal STRAIGHTS being extended,
the angles under the base
will be equal to one another;
—just what it was necessary to show.

ὅλη τῆ ὑπὸ AGZ γωνία
ἐδείχθη ἴση,
ὧν ἡ ὑπὸ GBH τῆ ὑπὸ BGZ
ἴση,
λοιπῆ ἄρα ἡ ὑπὸ ABF
λοιπῆ τῆ ὑπὸ AGB
ἐστὶν ἴση·
καὶ εἰσι πρὸς τῆ βάσει
τοῦ ABG τριγώνου.
ἐδείχθη δὲ καὶ
ἡ ὑπὸ ZBF τῆ ὑπὸ HGB ἴση·
καὶ εἰσιν ὑπὸ τὴν βάσιν.

Τῶν ἰσοσκελῶν τριγώνων
αἱ πρὸς τῆ βάσει γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ
προσεκβληθειῶν τῶν ἴσων εὐθειῶν
αἱ ὑπὸ τὴν βάσιν γωνίαι
ἴσαι ἀλλήλαις ἔσσονται·
ὅπερ ἔδει δεῖξαι.

ve bunlar ABG üçgeninin tabanıdır.
Ve ZBF açısının eşit olduğu göster-
ilmişti HGB açısına;
ve bunlar tabanın altındadır.

Dolayısıyla bir ikizkenar üçgenin ta-
banındaki açılar birbirine eşit-
tir,
ve, eşit doğrular uzatıldığında,
tabanın altında kalan açılar birbirine
eşit olacaklar;
—gösterilmesi gereken tam buydu.



1.6

If in a triangle
two angles be equal to one another,
also the sides that subtend the equal
angles
will be equal to one another.

Let there be
a triangle, ABG ,
having equal
angle ABG
to angle AGB .

I say that
also side AB to side AG
is equal.

For if unequal is AB to AG ,
one of them is greater.
Suppose AB be greater,
and there has been taken away

Ἐὰν τριγώνου
αἱ δύο γωνίαι ἴσαι ἀλλήλαις ᾖσιν,
καὶ αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι
πλευραὶ
ἴσαι ἀλλήλαις ἔσσονται.

Ἐστω
τρίγωνον τὸ ABG
ἴσην ἔχον
τὴν ὑπὸ ABG γωνίαν
τῆ ὑπὸ AGB γωνία·

λέγω, ὅτι
καὶ πλευρὰ ἡ AB πλευρᾶ τῆ AG
ἐστὶν ἴση.

Εἰ γὰρ ἀνίσος ἐστὶν ἡ AB τῆ AG ,
ἡ ἑτέρα αὐτῶν μείζων ἐστίν.
ἔστω μείζων ἡ AB ,
καὶ ἀφῆρησθῶ

Eğer bir üçgende
birbirine eşit iki açı varsa,
eşit açılardan gördüğümüz kenarlar da
birbirine eşit olacaklar.

Verilmiş olsun,
bir ABG üçgeni,
 ABG açısı eşit olan
 AGB açısına.

İddia ediyorum ki
 AB kenarı da AG kenarına
eşittir.

Çünkü eğer AB eşit değil ise AG ke-
narına,
biri daha büyüktür.
 AB daha büyük olan olsun,

from the greater, AB,
to the less, AG,
an equal, ΔB,
and there has been joined ΔΓ.

Since then ΔB is equal to AG,
and BΓ is common,
so the two ΔB and BΓ
to the two AG and BΓ
are equal,
either to either,
and angle ΔBΓ
to angle AΓB
is equal;
therefore the base ΔΓ to the base AB
is equal,
and triangle ΔBΓ to triangle AΓB
will be equal,
the less to the greater;
which is absurd.
therefore AB is not unequal to AG;
therefore it is equal.

If therefore in a triangle
two angles be equal to one another,
also the sides that subtend the equal
angles
will be equal to one another;
—just what it was necessary to show.

ἀπὸ τῆς μείζονος τῆς AB
τῆ ἐλάττοσι τῆ AG
ἴση ἢ ΔB,
καὶ ἐπεξεύχθω ἢ ΔΓ.

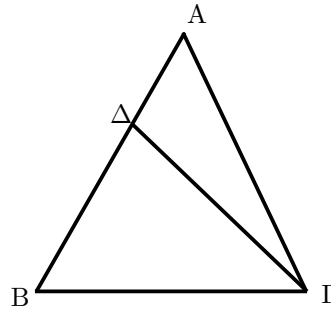
Ἐπεὶ οὖν ἴση ἐστὶν ἢ ΔB τῆ AG
κοινὴ δὲ ἢ BΓ,
δύο δὲ αἱ ΔB, BΓ
δύο ταῖς AG, ΓB
ἴσαι εἰσὶν
ἐκατέρα ἐκατέρα,
καὶ γωνία ἢ ὑπὸ ΔBΓ
γωνία τῆ ὑπὸ AΓB
ἐστὶν ἴση·
βάσις ἄρα ἢ ΔΓ βάσει τῆ AB
ἴση ἐστὶν,
καὶ τὸ ΔBΓ τρίγωνον τῷ AΓB τριγώνῳ
ἴσον ἐστὶν,
τὸ ἐλάσσον τῷ μείζονι·
ὅπερ ἄτοπον·
οὐκ ἄρα ἀνίσος ἐστὶν ἢ AB τῆ AG·
ἴση ἄρα.

Ἐὰν τριγώνου
αἱ δύο γωνίαι ἴσαι ἀλλήλαις ᾖσιν,
καὶ αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι
πλευραὶ
ἴσαι ἀλλήλαις ἔσονται·
ὅπερ ἔδει δεῖξαι.

ve diyelim, daha küçük olan AG ke-
narına eşit olan, ΔB,
daha büyük olan, AB kenarından ke-
silmiş olsun,
ve ΔΓ birleştirilmiş olsun.

O zaman ΔB eşittir AG kenarına,
ve BΓ ortaktır,
böylece ΔB, BΓ ikilisi eşittirler AG,
BΓ ikilisinin,
her biri birine,
ve ΔBΓ açısı eşittir AΓB açısına;
dolayısıyla ΔΓ tabanı eşittir AB ta-
banına,
ve ΔBΓ üçgeni eşit olacak AΓB üçge-
nine,
daha küçük daha büyüğe;
ki bu saçmadır.
dolayısıyla AB değildir eşit değil AG
kenarına;
dolayısıyla eşittir.

Dolayısıyla eğer bir üçgenin birbirine
eşit iki açısı varsa,
eşit açılardan karşıya gelen kenarlar eşittir;
—gösterilmesi gereken tam buydu.



1.7

On the same STRAIGHT,
to the same two STRAIGHTS,
two other STRAIGHTS,
[which are] equal,
either to either,
will not be constructed
to one and another point,¹
to the same parts,²
having the same extremities
as³ the original lines.

For if possible,
on the same STRAIGHT AB

Ἐπὶ τῆς αὐτῆς εὐθείας
δύο ταῖς αὐταῖς εὐθείαις
ἄλλαι δύο εὐθεῖαι
ἴσαι
ἐκατέρα ἐκατέρα
οὐ συσταθήσονται
πρὸς ἄλλω καὶ ἄλλω σημείῳ
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι
ταῖς ἐξ ἀρχῆς εὐθείαις.

Εἰ γὰρ δυνατόν,
ἐπὶ τῆς αὐτῆς εὐθείας τῆς AB

Aynı doğru üzerinde,
verilmiş iki doğruya,
eşit iki başka doğru,
her biri birine,
inşa edilmeyecek
bir ve başka bir noktaya,
aynı tarafta,
aynı uçları olan
başlangıçtaki doğrularla.

Çünkü eğer mümkünse,
aynı AB doğrusunda

¹Literally ‘another and another point’; more clearly in English, ‘to different points’.

²In English as apparently in Greek, *parts* can mean ‘region’—in this case, more precisely, ‘side’.

³According to Fowler ([5, as 8, p. 34] and [4, as 9, p. 38]), ‘As

is never to be regarded as a preposition’. This is unfortunate, since it means that the two constructions ‘Equal to X’ and ‘Same as X’ are not grammatically parallel. (We have ‘equal to him’, but ‘same as he’.) The constructions are parallel in Greek: ἴσος + DATIVE and αὐτός + DATIVE.

to two given STRAIGHTS $ΑΓ$, $ΓΒ$,
two other STRAIGHTS $ΑΔ$, $ΔΒ$,
equal
either to either
suppose have been constructed⁴
to one and another point
 $Γ$ and $Δ$,
to the same parts,
having the same extremities,
so that $ΓΑ$ is⁵ equal to $ΔΑ$,
having the same extremity as it, A ,
and $ΓΒ$ to $ΔΒ$,
having the same extremity as it, B ,
and suppose there has been joined
 $ΓΔ$.

Because equal is $ΑΓ$ to $ΑΔ$,
equal is
also angle $ΑΓΔ$ to $ΑΔΓ$;
Greater therefore [is]
 $ΑΔΓ$ than⁶ $ΔΓΒ$;⁷
by much, therefore, [is]
 $ΓΔΒ$ greater than $ΔΓΒ$.
Moreover, since equal is $ΓΒ$ to $ΔΒ$,
equal is also
angle $ΓΔΒ$ to angle $ΔΓΒ$.
But it was also shown than it
much greater;
which is absurd.

Not, therefore,
on the same STRAIGHT,
to the same two STRAIGHTS,
two other STRAIGHTS
[which are] equal,
either to either,
will be constructed
to one and another point
to the same parts
having the same extremities
as the original lines;
—just what it was necessary to show.

δύο ταῖς αὐταῖς εὐθείαις ταῖς $ΑΓ$, $ΓΒ$
ἄλλαι δύο εὐθεῖαι αἱ $ΑΔ$, $ΔΒ$
ἴσαι
ἐκατέρα ἐκατέρα
συνεστάτωσαν
πρὸς ἄλλῃ καὶ ἄλλῃ σημείῳ
τῷ τε $Γ$ καὶ $Δ$
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι,
ὥστε ἴσην εἶναι τὴν μὲν $ΓΑ$ τῇ $ΔΑ$
τὸ αὐτὸ πέρασ ἔχουσαν αὐτῇ τὸ A ,
τὴν δὲ $ΓΒ$ τῇ $ΔΒ$
τὸ αὐτὸ πέρασ ἔχουσαν αὐτῇ τὸ B ,
καὶ ἐπεζεύχθω
ἡ $ΓΔ$.

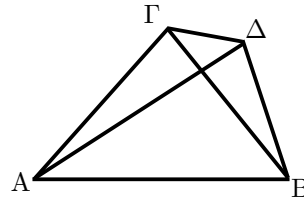
Ἐπει οὖν ἴση ἐστὶν ἡ $ΑΓ$ τῇ $ΑΔ$,
ἴση ἐστὶ
καὶ γωνία ἡ ὑπὸ $ΑΓΔ$ τῇ ὑπὸ $ΑΔΓ$.
μεῖζων ἄρα
ἡ ὑπὸ $ΑΔΓ$ τῆς ὑπὸ $ΔΓΒ$.
πολλῷ ἄρα
ἡ ὑπὸ $ΓΔΒ$ μεῖζων ἐστὶ τῆς ὑπὸ $ΔΓΒ$.
πάλιν ἐπει ἴση ἐστὶν ἡ $ΓΒ$ τῇ $ΔΒ$,
ἴση ἐστὶ καὶ
γωνία ἡ ὑπὸ $ΓΔΒ$ γωνία τῇ ὑπὸ $ΔΓΒ$.
ἐδείχθη δὲ αὐτῆς καὶ
πολλῷ μεῖζων.
ὅπερ ἐστὶν ἀδύνατον.

Οὐκ ἄρα
ἐπὶ τῆς αὐτῆς εὐθείας
δύο ταῖς αὐταῖς εὐθείαις
ἄλλαι δύο εὐθεῖαι
ἴσαι
ἐκατέρα ἐκατέρα
συσταθήσονται
πρὸς ἄλλῃ καὶ ἄλλῃ σημείῳ
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι
ταῖς ἐξ ἀρχῆς εὐθείαις.
ὅπερ ἔδει δεῖξαι.

verilmiş iki $ΑΓ$, $ΓΒ$ doğrusuna
eşit başka iki $ΑΔ$, $ΔΒ$ doğrusu
her biri birine
—diyelim inşa edilmiş olsunlar
bir ve başka bir noktaya
 $Γ$ ve $Δ$,
aynı tarafta,
aynı uçları olan,
şöyle ki $ΓΑ$ eşit olmalı $ΔΑ$ doğrusuna,
aynı A ucuna sahip olan,
ve $ΓΒ$, $ΔΒ$ doğrusuna,
aynı B ucuna sahip olan,
ve $ΓΔ$ birleştirilmiş olsun.

Çünkü $ΑΓ$ eşittir $ΑΔ$ doğrusuna,
yine eşittir
 $ΑΓΔ$, $ΑΔΓ$ açısına;
dolayısıyla $ΑΔΓ$ büyüktür $ΔΓΒ$
açısından;
dolayısıyla $ΓΔΒ$ çok daha büyüktür
 $ΔΓΒ$ açısından.
Üstelik $ΓΒ$ eşit olduğu için $ΔΒ$
doğrusuna,
 $ΓΔΒ$ açısı eşittir $ΔΓΒ$ açısına.
Ama ondan çok daha büyük olduğu
gösterilmişti;
ki bu saçmadır.

Şöyle olmaz, dolayısıyla; aynı doğru
üzerinde,
verilmiş iki doğruya,
iki başka doğru, eşit,
her biri birine,
inşa edilecek
başka bir noktaya
aynı tarafta
aynı uçları olan
başlangıçtaki doğrularla.
—gösterilmesi gereken tam buydu.



⁴The Perseus Project Word Study Tool does not recognize $συνεστάτωσαν$ here, but it should be just the plural form of $συνεστάτω$, which is used for example in Proposition I.2 and which Perseus declares to be a passive perfect imperative. The active third-person imperative ending $-τωσαν$ (instead of the older $-των$) is said by Smyth [16, 466] to appear in prose after Thucydides. This describes Euclid. However, I cannot explain from Smyth the use of an active perfect (as opposed to aorist) form with passive meaning. Presumably the verb is used 'impersonally'. The LSJ lexicon [10] cites the

present proposition under $συνίστημι$. See also the note at I.21.

⁵The Greek verb is an infinitive. An infinitive clause may follow ὥστε [16, ¶2260, p. 507]. Compare the enunciation of Proposition 1.

⁶Fowler ([5, **than 6**, p. 629] and [4, **than 6**, p. 619]) does grant the possibility of construing 'than' as a preposition, though he disapproves. Then English cannot exactly mirror the Greek $μεῖζων$ + GENITIVE. Turkish does mirror it with $-den$ büyük. See note 3 above.

⁷Here one must refer to the diagram.

1.8

If two triangles
two sides
to two sides
have equal,
either to either,
and have also base equal to base,
also angle to angle
they will have equal,
[namely] that by the equal STRAIGHTS
subtended.

Let there be
two triangles, $AB\Gamma$ and ΔEZ ,
the two sides AB and $A\Gamma$
to the two sides ΔE and ΔZ
having equal,
either to either,
 AB to ΔE ,
and $A\Gamma$ to ΔZ ;
and let them have
base $B\Gamma$ equal to base EZ .

I say that
also angle BAG
to angle $E\Delta Z$
is equal.

For, there being applied
triangle $AB\Gamma$
to triangle ΔEZ ,
and there being placed
the point B on the point E ,
and the STRAIGHT $B\Gamma$ on EZ ,
also the point Γ will apply to Z ,
by the equality of $B\Gamma$ to EZ .
Then, $B\Gamma$ applying to EZ ,
also will apply
 BA and ΓA to $E\Delta$ and ΔZ .
For if base $B\Gamma$ to the base EZ
apply,
and sides BA , $A\Gamma$ to $E\Delta$, ΔZ
do not apply,
but deviate,
as EH , HZ ,
there will be constructed
on the same STRAIGHT,
to two given STRAIGHTS,
two other STRAIGHTS equal,
either to either,
to one and another point
to the same parts
having the same extremities.
But they are not constructed;
therefore it is not [the case] that,
there being applied
the base $B\Gamma$ to the base EZ ,
there do not apply
sides BA , $A\Gamma$ to $E\Delta$, ΔZ .
Therefore they apply.
So angle BAG

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχῃ
ἐκατέραν ἐκατέρα,
ἔχη δὲ καὶ τὴν βάσιν τῆς βάσει ἴσην,
καὶ τὴν γωνίαν τῆς γωνία
ἴσην ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην.

Ἔστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , $A\Gamma$
ταῖς δύο πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἐκατέραν ἐκατέρα,
τὴν μὲν AB τῆς ΔE
τὴν δὲ $A\Gamma$ τῆς ΔZ .
ἔχέτω δὲ
καὶ βάσιν τὴν $B\Gamma$ βάσει τῆς EZ ἴσην.

λέγω, ὅτι
καὶ γωνία ἡ ὑπὸ BAG
γωνία τῆς ὑπὸ $E\Delta Z$
ἴσος ἐστίν.

Ἐφαρμοζομένου γὰρ
τοῦ $AB\Gamma$ τριγώνου
ἐπὶ τὸ ΔEZ τρίγωνον
καὶ τιθεμένου
τοῦ μὲν B σημείου ἐπὶ τὸ E σημεῖον
τῆς δὲ $B\Gamma$ εὐθείας ἐπὶ τὴν EZ
ἐφαρμόσει καὶ τὸ Γ σημεῖον ἐπὶ τὸ Z
διὰ τὸ ἴσην εἶναι τὴν $B\Gamma$ τῆς EZ .
ἐφαρμοσάσης δὲ τῆς $B\Gamma$ ἐπὶ τὴν EZ
ἐφαρμόσουσι καὶ
αἱ BA , ΓA ἐπὶ τὰς $E\Delta$, ΔZ .
εἰ γὰρ βάσις μὲν ἡ $B\Gamma$ ἐπὶ βάσιν τὴν EZ
ἐφαρμόσει,
αἱ δὲ BA , $A\Gamma$ πλευραὶ ἐπὶ τὰς $E\Delta$, ΔZ
οὐκ ἐφαρμόσουσιν
ἀλλὰ παραλλάξουσιν
ὡς αἱ EH , HZ ,
συσταθήσονται
ἐπὶ τῆς αὐτῆς εὐθείας
δύο ταῖς αὐταῖς εὐθείαις
ἄλλαι δύο εὐθεῖαι ἴσαι
ἐκατέρα ἐκατέρα
πρὸς ἄλλω καὶ ἄλλω σημείω
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι.
οὐ συνίστανται δὲ
οὐκ ἄρα
ἐφαρμοζομένης
τῆς $B\Gamma$ βάσεως ἐπὶ τὴν EZ βάσιν
οὐκ ἐφαρμόσουσι
καὶ αἱ BA , $A\Gamma$ πλευραὶ ἐπὶ τὰς $E\Delta$, ΔZ .
ἐφαρμόσουσιν ἄρα
ὥστε καὶ γωνία ἡ ὑπὸ BAG

Eğer iki üçgenin, varsa iki kenarı eşit
olan iki kenara,
her bir (kenar) birine,
ve varsa tabana eşit tabanı,
ayrıca olacak açığa eşit açıları,
(yani) eşit kenarları görenler.

Verilmiş olsun
iki üçgen, $AB\Gamma$ ve ΔEZ ,
iki kenarı AB , $A\Gamma$ eşit olan ΔE , ΔZ
iki kenarının
her biri birine,
 AB , ΔE kenarına,
ve $A\Gamma$, ΔZ kenarına;
ve onların
 $B\Gamma$ tabanı eşit olsun EZ tabanına.

İddia ediyorum ki
 BAG açısı da
eşittir $E\Delta Z$ açısına.

Çünkü, üstüne koyulursa
 $AB\Gamma$ üçgeni ΔEZ üçgeninin,
ve yerleştirilirse
 B noktası E noktasına,
ve $B\Gamma$, EZ doğrusuna,
 Γ noktası da yerleşecek Z noktasına,
sayesinde eşitliğinin $B\Gamma$ doğrusunun
 EZ doğrusuna.
O zaman, $B\Gamma$ yerleştirilince EZ
doğrusuna,
 BA ve ΓA doğruları da yerleşecekler
 $E\Delta$ ve ΔZ doğrularına.
Çünkü eğer $B\Gamma$ yerleşirse EZ ta-
banına,
ve BA , $A\Gamma$ kenarları yerleşmezse $E\Delta$,
 ΔZ kenarlarına,
ama sapsarsa,
 EH ve HZ olarak
inşa edilmiş olacak
aynı doğru üzerinde,
verilmiş iki doğruya,
iki başka doğru eşit,
her biri birine,
başka bir noktaya
aynı tarafta
aynı uçları olan.
Ama inşa edilmediler;
dolayısıyla (durum) şöyle değil,
 $B\Gamma$ tabanı yerleştirilince EZ tabanına,
 BA , $A\Gamma$ kenarları yerleşmez $E\Delta$, ΔZ
kenarlarına.
Dolayısıyla yerleşirler.
Böylece BAG açısı yerleşecek $E\Delta Z$

to angle $E\Delta Z$
will apply
and will be equal to it.

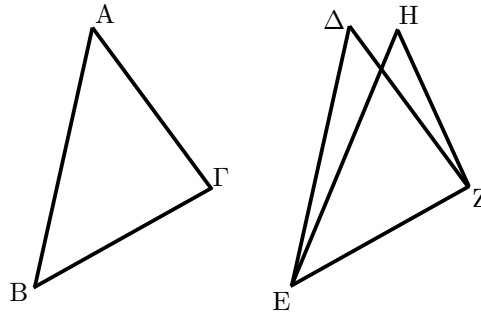
If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
and have also base equal to base,
also angle to angle
they will have equal,
[namely] that by the equal STRAIGHTS
subtended;
—just what it was necessary to show.

ἐπὶ γωνίαν τὴν ὑπὸ $E\Delta Z$
ἐφαρμόσει
καὶ ἴση αὐτῆ ἔσται.

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα,
ἔχη δὲ καὶ τὴν βάσιν τῆ βάσει ἴσην,
καὶ τὴν γωνίαν τῆ γωνία
ἴσην ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην·
ὅπερ ἔδει δεῖξαι.

açısına
ve ona eşit olacak.

Eğer, dolayısıyla, iki üçgenin,
varsa iki kenarı
eşit olan
iki kenara,
her bir (kenar) birine,
ve varsa tabana eşit tabanı,
ayrıca olacak açıya eşit açıları,
(yani) eşit kenarları görenler;
—gösterilmesi gereken tam buydu.



1.9

The¹ given rectilinear angle
to cut in two.²

Let be
the given rectilinear angle
 $B\Delta\Gamma$.

Then it is necessary
to cut it in two.

Suppose there has been chosen
on AB at random a point Δ ,
and there has been taken from $A\Gamma$
 AE , equal to $A\Delta$,
and ΔE has been joined,
and there has been constructed on ΔE
an equilateral triangle, ΔEZ ,
and AZ has been joined.

I say that
angle $B\Delta\Gamma$ has been cut in two
by the STRAIGHT AZ .
For, because $A\Delta$ is equal to AE ,
and AZ is common,
then the two, ΔA and AZ
to the two, EA and AZ ,
are equal,

Τὴν δοθεῖσαν γωνίαν εὐθύγραμμον
δίχα τεμεῖν.

Ἐστω
ἡ δοθεῖσα γωνία εὐθύγραμμος
ἡ ὑπὸ $B\Delta\Gamma$.

δεῖ δὴ
αὐτὴν δίχα τεμεῖν.

Εἰλήφθω
ἐπὶ τῆς AB τυχὸν σημείον τὸ Δ ,
καὶ ἀφηρήσθω ἀπὸ τῆς $A\Gamma$
τῆ $A\Delta$ ἴση ἡ AE ,
καὶ ἐπεζεύχθω ἡ ΔE ,
καὶ συνεστάτω ἐπὶ τῆς ΔE
τρίγωνον ἰσόπλευρον τὸ ΔEZ ,
καὶ ἐπεζεύχθω ἡ AZ .

λέγω, ὅτι
ἡ ὑπὸ $B\Delta\Gamma$ γωνία δίχα τέτμηται
ὑπὸ τῆς AZ εὐθείας.
Ἐπεὶ γὰρ ἴση ἐστὶν ἡ $A\Delta$ τῆ AE ,
κοινὴ δὲ ἡ AZ ,
δύο δὴ αἱ ΔA , AZ
δυσὶ ταῖς EA , AZ
ἴσαι εἰσὶν

Verilen düzkenar açıyı
ikiye kesmek.

Verilmiş olsun
düzkenar bir açı, $B\Delta\Gamma$.

Şimdi gereklidir
onun ikiye kesilmesi.

Diyelim seçilmiş olsun
 AB üzerinde rastgele bir nokta, Δ ,
ve kesilmiş olsun $A\Gamma$ doğrusundan
 AE , eşit olan $A\Delta$ doğrusuna,
ve ΔE birleştirilmiş olsun,
ve inşa edilmiş olsun ΔE üzerinde
bir eşkenar üçgen, ΔEZ ,
ve AZ birleştirilmiş olsun.

İddia ediyorum ki
 $B\Delta\Gamma$ açısı ikiye kesilmiş oldu
 AZ doğrusu tarafından.
Çünkü, olduğundan, $A\Delta$ eşit AE ke-
narına,
ve AZ ortak,
 ΔA , AZ ikilisi eşittirler EA , AZ ikil-
isinin

¹Here the generic article (see note 1 to Proposition 1 above) is particularly appropriate. Suppose we take a straight line with a point A on it and draw a circle with center A cutting the line at B and C . Then the straight line BC has been bisected at A . In particular, a line has been bisected. But this does not mean we have solved the problem of the present proposition. In modern mathemat-

ical English, the proposition could indeed be 'To bisect a rectilinear angle'; but then 'a' must be understood as 'an arbitrary' or 'a given'. Of course, Euclid does supply this qualification in any case.

²For 'cut in two' we could say 'bisect'; but in at least one place, in Proposition 12, $\delta\acute{\iota}\chi\alpha$ $\tau\epsilon\mu\epsilon\acute{\iota}\nu$ will be separated.

either to either,
and the base ΔZ to the base EZ
is equal;
therefore angle ΔAZ
to angle EAZ
is equal.

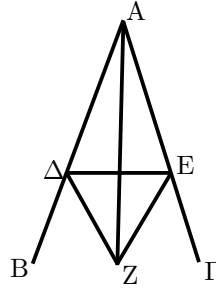
Therefore the given rectilinear angle
 BAG
has been cut in two
by the STRAIGHT AZ ;
—just what it was necessary to do.

ἑκατέρα ἑκατέρα.
καὶ βάσις ἡ ΔZ βάσει τῆς EZ
ἴση ἐστίν.
γωνία ἄρα ἡ ὑπὸ ΔAZ
γωνία τῆς ὑπὸ EAZ
ἴση ἐστίν.

Ἡ ἄρα δοθεῖσα γωνία εὐθύγραμμος
ἡ ὑπὸ BAG
δίχα τέτμηται
ὑπὸ τῆς AZ εὐθείας.
ὅπερ ἔδει ποιῆσαι.

her biri birine ,
ve ΔZ tabanı EZ tabanına eşittir;
dolayısıyla ΔAZ açısı EAZ açısına
eşittir.

Dolayısıyla verilen düzkenar açı BAG
kesilmiş oldu ikiye
 AZ doğrusunca;
—yapılması gereken tam buydu.



1.10

The given bounded STRAIGHT
to cut in two.

Let be
the given bounded straight line AB .

It is necessary then
the bounded straight line AB to cut
in two.

Suppose there has been constructed
on it
an equilateral triangle, ABG ,
and suppose has been cut in two
the angle AGB by the STRAIGHT $\Gamma\Delta$.

I say that
the STRAIGHT AB has been cut in two
at the point Δ .
For, because AG is equal to AB ,
and $\Gamma\Delta$ is common,
the two, AG and $\Gamma\Delta$,
to the two, BG , $\Gamma\Delta$,
are equal,
either to either,
and angle $AG\Delta$
to angle $BG\Delta$
is equal;
therefore the base $A\Delta$ to the base $B\Delta$
is equal.

Therefore the given bounded
STRAIGHT,
 AB ,
has been cut in two at Δ ;
—just what it was necessary to do.

Τὴν δοθεῖσαν εὐθεῖαν πεπερασμένην
δίχα τεμεῖν.

Ἐστω
ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ AB .

δεῖ δὴ
τὴν AB εὐθεῖαν πεπερασμένην δίχα
τεμεῖν.

Συνεστάτω ἐπ' αὐτῆς
τρίγωνον ἰσόπλευρον τὸ ABG ,
καὶ τεμήσθω
ἡ ὑπὸ AGB γωνία δίχα τῆς $\Gamma\Delta$ εὐθείας.

λέγω, ὅτι
ἡ AB εὐθεῖα δίχα τέτμηται
κατὰ τὸ Δ σημείον.
Ἐπεὶ γὰρ ἴση ἐστίν ἡ AG τῆς GB ,
κοινὴ δὲ ἡ $\Gamma\Delta$,
δύο δὲ αἱ AG , $\Gamma\Delta$
δύο ταῖς BG , $\Gamma\Delta$
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρα.
καὶ γωνία ἡ ὑπὸ $AG\Delta$
γωνία τῆς ὑπὸ $BG\Delta$
ἴση ἐστίν.
βάσις ἄρα ἡ $A\Delta$ βάσει τῆς $B\Delta$
ἴση ἐστίν.

Ἡ ἄρα δοθεῖσα εὐθεῖα πεπερασμένη
ἡ AB
δίχα τέτμηται κατὰ τὸ Δ .
ὅπερ ἔδει ποιῆσαι.

Verilen sınırlı doğruyu
ikiye kesmek.

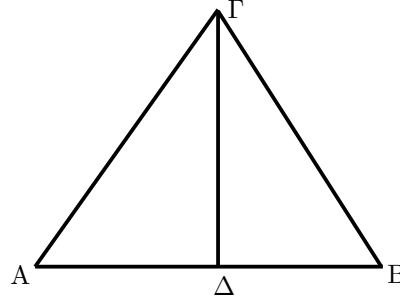
Verilmiş olsun
bir sınırlı doğru, AB .

Gereklidir,
verilmiş AB sınırlı doğrusunu, kesmek
ikiye.

Kabul edelim ki üzerinde inşa edilmiş
olsun
bir eşkenar üçgen, ABG ,
ve AGB açısı kesilmiş olsun ikiye
 $\Gamma\Delta$ doğrusunca.

İddia ediyorum ki
 AB doğrusu ikiye kesilmiş oldu
 Δ noktasında. Çünkü, AG eşit
olduğundan AB kenarına,
ve $\Gamma\Delta$ ortak,
 AG ve $\Gamma\Delta$ ikilisi, eşittirler BG , $\Gamma\Delta$ ik-
ilisinin,
her biri birine,
ve $AG\Delta$ açısı eşittir $BG\Delta$ açısına;
dolayısıyla $A\Delta$ tabanı, $B\Delta$ tabanına,
eşittir.

Dolayısıyla verilmiş sınırlı AB
doğrusu
 Δ noktasında ikiye kesilmiş oldu;
—yapılması gereken tam buydu.



1.11

To the given STRAIGHT
from the given point on it
at right angles
to draw¹ a straight line.²

Let be
the given STRAIGHT AB,
and the given point on it, Γ.

It is necessary then
from the point Γ
to the STRAIGHT AB
at right angles
to draw a straight line.

Suppose there has been chosen
on AΓ at random a point Δ,
and there has been laid down
an equal to ΓΔ, [namely] ΓΕ,
and there has been constructed
on ΔΕ
an equilateral triangle, ΖΔΕ,
and there has been joined ΖΓ.

I say that
to the given straight line AB
from the given point on it,
Γ,
at right angles
has been drawn a straight line, ΖΓ.
For, since ΔΓ is equal to ΓΕ,
and ΓΖ is common,
the two, ΔΓ and ΓΖ,
to the two, ΕΓ and ΓΖ,
are equal,
either to either;
and the base ΔΖ to the base ΖΕ
is equal;
therefore angle ΔΓΖ
to angle ΕΓΖ
is equal;
and they are adjacent.
Whenever a STRAIGHT,
standing on a STRAIGHT,
the adjacent angles
equal to one another
make,

Τῆ δοθείσῃ εὐθείᾳ
ἀπὸ τοῦ πρὸς αὐτῇ δοθέντος σημείου
πρὸς ὀρθὰς γωνίας
εὐθεῖαν γραμμὴν ἀγαγεῖν.

Ἐστω
ἡ μὲν δοθεῖσα εὐθεῖα ἡ ΑΒ
τὸ δὲ δοθὲν σημεῖον ἐπ' αὐτῆς τὸ Γ.

δεῖ δὴ
ἀπὸ τοῦ Γ σημείου
τῆ ΑΒ εὐθείᾳ
πρὸς ὀρθὰς γωνίας
εὐθεῖαν γραμμὴν ἀγαγεῖν.

Εἰλήφθω
ἐπὶ τῆς ΑΓ τυχὸν σημεῖον τὸ Δ,
καὶ κείσθω
τῆ ΓΔ ἴση ἡ ΓΕ,
καὶ συνεστάτω
ἐπὶ τῆς ΔΕ τρίγωνον ἰσόπλευρον
τὸ ΖΔΕ,
καὶ ἐπεζεύχθω ἡ ΖΓ.

λέγω, ὅτι
τῆ δοθείσῃ εὐθείᾳ τῆ ΑΒ
ἀπὸ τοῦ πρὸς αὐτῇ δοθέντος σημείου
τοῦ Γ
πρὸς ὀρθὰς γωνίας
εὐθεῖα γραμμὴ ἤχται ἡ ΖΓ.
Ἐπεὶ γὰρ ἴση ἐστὶν ἡ ΔΓ τῆ ΓΕ,
κοινὴ δὲ ἡ ΓΖ,
δύο δὴ αἱ ΔΓ, ΓΖ
δυσὶ ταῖς ΕΓ, ΓΖ
ἴσαι εἰσὶν
ἐκατέρα ἐκατέρᾳ·
καὶ βάσις ἡ ΔΖ βάσει τῆ ΖΕ
ἴση ἐστὶν·
γωνία ἄρα ἡ ὑπὸ ΔΓΖ
γωνία τῆ ὑπὸ ΕΓΖ
ἴση ἐστὶν·
καὶ εἰσὶν ἐφεξῆς.
ὅταν δὲ εὐθεῖα
ἐπ' εὐθεῖαν σταθεῖσα
τὰς ἐφεξῆς γωνίας
ἴσας ἀλλήλαις
ποιῇ,

Verilen bir doğruya
üzerinde verilen bir noktada
dik açılarda
bir doğru çizmek.

Verilmiş olsun
bir doğru, ΑΒ,
ve üzerinde bir nokta, Γ.

Gereklidir
Γ noktasında
ΑΒ doğrusuna
dik açılarda
bir doğru.

Kabul edelim ki seçilmiş olsun
ΑΓ doğrusunda rastgele bir nokta, Δ,
ve yerleştirilmiş olsun
ΓΕ eşit olarak ΓΔ doğrusuna,
ve inşa edilmiş olsun
ΔΕ üzerinde bir eşkenar üçgen, ΖΔΕ,
ve ΖΓ birleştirilmiş olsun.

İddia ediyorum ki
verilen ΑΒ doğrusuna
üzerindeki Γ noktasında
dik açılarda
bir ΖΓ doğrusu çizilmiş oldu.
Çünkü, ΔΓ eşit olduğundan ΓΕ
doğrusuna,
ve ΓΖ ortak olduğundan,
ΔΓ ve ΓΖ ikilisi,
eşittirler ΕΓ ve ΓΖ ikilisinin,
her biri birine;
ve ΔΖ tabanı eşittir ΖΕ tabanına;
dolayısıyla ΔΓΖ açısı eşittir ΕΓΖ
açısına;
ve bitişiktirler.
Ne zaman bir doğru,
bir doğru üzerinde dikilen,
bitişik açıları birbirine eşit yaparsa,
bu açının her biri dik olur.
Dolayısıyla ΔΓΖ, ΖΓΕ açılarının her
ikisi de diktir.

¹This is the first time among the propositions that Euclid writes
out *straight line* (εὐθεῖα γραμμὴ) and not just *straight* (εὐθεῖα).

²Literally 'lead, conduct'.

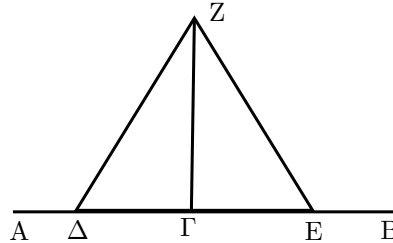
either of the equal angles is right.
Right therefore is either of the angles $\Delta\Gamma Z$ and $Z\Gamma E$.

Therefore, to the given STRAIGHT AB, from the given point on it, Γ , at right angles, has been drawn the straight line ΓZ ;—just what it was necessary to do.

ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστίν·
ὀρθὴ ἄρα ἐστὶν ἑκατέρα τῶν ὑπὸ $\Delta\Gamma Z$, $Z\Gamma E$.

Τῆ ἄρα δοθείσῃ εὐθείᾳ τῆ AB ἀπὸ τοῦ πρὸς αὐτῇ δοθέντος σημείου τοῦ Γ πρὸς ὀρθὰς γωνίας εὐθεῖα γραμμὴ ἤχεται ἡ ΓZ · ὅπερ ἔδει ποιῆσαι.

Dolayısıyla, verilen AB doğrusuna, üzerinde verilmiş Γ noktasında, dik açılarda, bir ΓZ doğrusu çizilmiş oldu;—yapılması gereken tam buydu.



1.12

To the given unbounded STRAIGHT, from the given point, which is not on it, to draw a perpendicular straight line.

Let be the given unbounded STRAIGHT AB, and the given point, which is not on it, Γ .

It is necessary then to the given unbounded STRAIGHT, AB from the given point Γ , which is not on it, to draw a perpendicular straight line.

For suppose there has been chosen on the other parts of the STRAIGHT AB at random a point Δ , and to the center Γ , at the distance $\Gamma\Delta$, a circle has been drawn, EZH, and has been cut the STRAIGHT EH in two at Θ , and there have been joined the STRAIGHTS ΓH , $\Gamma\Theta$, and ΓE .

I say that to the given unbounded STRAIGHT AB, from the given point Γ , which is not on it, has been drawn a perpendicular, $\Gamma\Theta$. For, because $H\Theta$ is equal to ΘE , and $\Theta\Gamma$ is common, the two, $H\Theta$ and $\Theta\Gamma$,

Ἐπὶ τὴν δοθείσαν εὐθεῖαν ἄπειρον ἀπὸ τοῦ δοθέντος σημείου, ὃ μὴ ἐστὶν ἐπ' αὐτῆς, κάθετον εὐθεῖαν γραμμὴν ἀγαγεῖν.

Ἐστω ἡ μὲν δοθείσα εὐθεῖα ἄπειρος ἡ AB τὸ δὲ δοθὲν σημεῖον, ὃ μὴ ἐστὶν ἐπ' αὐτῆς, τὸ Γ .

δεῖ δὴ ἐπὶ τὴν δοθείσαν εὐθεῖαν ἄπειρον τὴν AB ἀπὸ τοῦ δοθέντος σημείου τοῦ Γ , ὃ μὴ ἐστὶν ἐπ' αὐτῆς, κάθετον εὐθεῖαν γραμμὴν ἀγαγεῖν.

Εἰλήφθω γὰρ ἐπὶ τὰ ἕτερα μέρη τῆς AB εὐθείας τυχὸν σημεῖον τὸ Δ , καὶ κέντρῳ μὲν τῷ Γ διαστήματι δὲ τῷ $\Gamma\Delta$ κύκλος γεγράφθω ὁ EZH, καὶ τετμήσθω ἡ EH εὐθεῖα δίχα κατὰ τὸ Θ , καὶ ἐπεζεύχθωσαν αἱ ΓH , $\Gamma\Theta$, ΓE εὐθεῖαι·

λέγω, ὅτι ἐπὶ τὴν δοθείσαν εὐθεῖαν ἄπειρον τὴν AB ἀπὸ τοῦ δοθέντος σημείου τοῦ Γ , ὃ μὴ ἐστὶν ἐπ' αὐτῆς, κάθετος ἤχεται ἡ $\Gamma\Theta$. Ἐπεὶ γὰρ ἴση ἐστὶν ἡ $H\Theta$ τῆ ΘE , κοινὴ δὲ ἡ $\Theta\Gamma$, δύο δὲ αἱ $H\Theta$, $\Theta\Gamma$

Verilen sınırlanmamış doğruya, verilen bir noktadan, üzerinde olmayan, bir dik doğru çizmek.

Verilmiş olsun bir sınırlanmamış doğru, AB, ve bir nokta, üzerinde olmayan, Γ .

Gereklidir verilmiş AB sınırlanmamış doğruya verilmiş Γ noktasından, üzerinde olmayan, bir dik doğru çizmek.

Çünkü kabul edelim ki seçilmiş olsun AB doğrusunun diğer tarafında rastgele bir Δ noktası, ve Γ merkezinde, $\Gamma\Delta$ uzaklığında, bir çember çizilmiş olsun, EZH, ve EH doğrusu Θ noktasında ikiye kesilmiş olsun, ve birleştirilmiş olsun ΓH , $\Gamma\Theta$, ve ΓE doğruları.

İddia ediyorum ki verilen sınırlanmamış AB doğruya, verilen Γ noktasından, üzerinde olmayan, çizilmiş oldu dik $\Gamma\Theta$ doğrusu. Çünkü, $H\Theta$ eşit olduğundan ΘE doğrusuna, ve $\Theta\Gamma$ ortak, $H\Theta$ ve $\Theta\Gamma$ ikilisi,

to the two, $E\Theta$ and $\Theta\Gamma$, are equal,
 either to either;
 and the base ΓH to the base ΓE
 is equal;
 therefore angle $\Gamma\Theta H$
 to angle $E\Theta\Gamma$
 is equal;
 and they are adjacent.
 Whenever a STRAIGHT,
 standing on a STRAIGHT,
 the adjacent angles
 equal to one another make,
 right
 either of the equal angles is,
 and
 the STRAIGHT that has been stood
 is called perpendicular
 to that on which it has been stood.

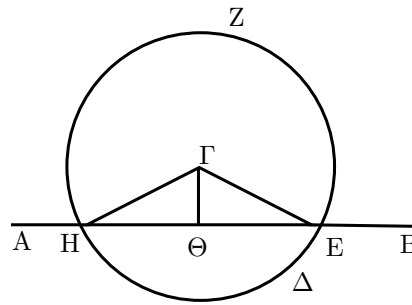
Therefore, to the given unbounded
 STRAIGHT AB ,
 from the given point Γ ,
 which is not on it,
 a perpendicular $\Gamma\Theta$ has been drawn;
 —just what it was necessary to do.

δύο ταῖς $E\Theta$, $\Theta\Gamma$ ἴσαι εἰσὶν
 ἑκάτερα ἑκατέρῃ
 καὶ βάσις ἢ ΓH βάσει τῇ ΓE
 ἔστιν ἴση·
 γωνία ἄρα ἢ ὑπὸ $\Gamma\Theta H$
 γωνία τῇ ὑπὸ $E\Theta\Gamma$
 ἔστιν ἴση,
 καὶ εἰσὶν ἐφελξῆς.
 ὅταν δὲ εὐθεῖα
 ἐπ' εὐθεῖαν σταθεῖσα
 τὰς ἐφελξῆς γωνίας
 ἴσας ἀλλήλαις ποιῇ,
 ὀρθὴ
 ἑκάτερα τῶν ἴσων γωνιῶν ἔστιν,
 καὶ
 ἢ ἐφελξῆς εὐθεῖα
 κάθετος καλεῖται
 ἐφ' ἣν ἐφέστηκεν.

Ἐπὶ τὴν δοθεῖσαν ἄρα εὐθεῖαν ἄπειρον
 τὴν AB
 ἀπὸ τοῦ δοθέντος σημείου τοῦ Γ ,
 ὃ μὴ ἐστὶν ἐπ' αὐτῆς,
 κάθετος ἤχεται ἢ $\Gamma\Theta$.
 ὅπερ ἔδει ποιῆσαι.

eşittirler $E\Theta$ ve $\Theta\Gamma$ ikilisinin,
 her biri birine;
 ve ΓH tabanı eşittir ΓE tabanına;
 dolayısıyla $\Gamma\Theta H$ açısı eşittir $E\Theta\Gamma$
 açısına.
 Ve onlar bitişiktirler.
 Ne zaman bir doğru,
 bir doğru üzerinde dikildiğinde,
 bitişik açıları birbirine eşit yaparsa,
 açılardan her biri eşittir,
 ve diktilen doğru
 üzerinde
 dikildiği doğruya diktir denir.

Dolayısıyla, verilen AB sınırlandırıl-
 mamış doğruya,
 verilen Γ noktasından,
 üzerinde olmayan,
 bir dik, $\Gamma\Theta$, çizilmiş oldu;
 —yapılması gereken tam buydu.



1.13

If a STRAIGHT,
 stood on a STRAIGHT,
 make angles,
 either two RIGHTS
 or equal to two RIGHTS
 it will make [them].

For, some STRAIGHT, AB ,
 stood on the STRAIGHT $\Gamma\Delta$,
 —suppose it makes¹ angles
 ΓBA and $AB\Delta$.

I say that
 the angles ΓBA and $AB\Delta$
 either are two RIGHTS
 or [are] equal to two RIGHTS.

If equal is
 ΓBA to $AB\Delta$,
 they are two RIGHTS.

If not,

Ἐὰν εὐθεῖα
 ἐπ' εὐθεῖαν σταθεῖσα
 γωνίας ποιῇ,
 ἤτοι δύο ὀρθὰς
 ἢ δυοῖν ὀρθαῖς ἴσας
 ποιήσει.

Εὐθεῖα γὰρ τις ἢ AB
 ἐπ' εὐθεῖαν τὴν $\Gamma\Delta$ σταθεῖσα
 γωνίας ποιείτω
 τὰς ὑπὸ ΓBA , $AB\Delta$.

λέγω, ὅτι
 αἱ ὑπὸ ΓBA , $AB\Delta$ γωνίαι
 ἤτοι δύο ὀρθαὶ εἰσὶν
 ἢ δυοῖν ὀρθαῖς ἴσαι.

Εἰ μὲν οὖν ἴση ἐστὶν
 ἢ ὑπὸ ΓBA τῇ ὑπὸ $AB\Delta$,
 δύο ὀρθαὶ εἰσὶν.

εἰ δὲ οὐ,

Eğer bir doğru,
 dikiltilmiş bir doğrunun üzerine,
 yaparsa açılar,
 ya iki dik
 ya da iki dike eşit
 yapacak (onları).

Çünkü, bir AB doğrusuda,
 dikiltilmiş $\Gamma\Delta$ doğrusu,
 —kabul edelim ki ΓBA ve $AB\Delta$
 açılarını oluşturmuş olsun.

İddia ediyorum ki
 ΓBA ve $AB\Delta$ açıları
 ya iki dik açıdır
 ya da iki dik açıya eşittir(ler).

Eğer ΓBA eşitse $AB\Delta$ açısına,
 iki dik açıdırlar.

Eğer değilse,

¹ Euclid uses a *present, active* imperative here.

suppose there has been drawn,
from the point B,
to the [STRAIGHT] ΓΔ,
at right angles,
BE.

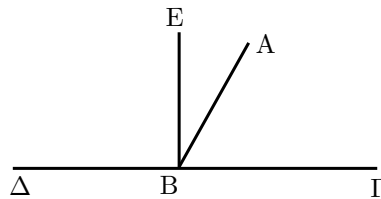
Therefore ΓBE and EBD
are two RIGHTS;
and since ΓBE
to the two, ΓBA and ABE, is equal
let there be added in common EBD.
Therefore ΓBE and EBD
to the three, ΓBA, ABE, and EBD,
are equal.
Moreover,
since ΔBA
to the two, ΔBE and EBA, is equal
let there be added in common ABΓ;
therefore ΔBA and ABΓ
to the three, ΔBE, EBA, and ABΓ,
are equal.
And ΓBE and EBD were shown
equal to the same three.
And equals to the same
are also equal to one another;
also, therefore, ΓBE and EBD
to ΔBA and ABΓ are equal;
but ΓBE and EBD
are two RIGHTS;
and therefore ΔBA and ABΓ
are equal to two RIGHTS.

If, therefore, a STRAIGHT,
stood on a STRAIGHT,
make angles,
either two RIGHTS
or equal to two RIGHTS
it will make;
—just what it was necessary to show.

ἤχθω
ἀπὸ τοῦ Β σημείου
τῆ ΓΔ [εὐθείᾳ]
πρὸς ὀρθὰς
ἢ ΒΕ·

αἱ ἄρα ὑπὸ ΓΒΕ, ΕΒΔ
δύο ὀρθαὶ εἰσίν·
καὶ ἐπεὶ ἡ ὑπὸ ΓΒΕ
δυοὶ ταῖς ὑπὸ ΓΒΑ, ΑΒΕ ἴση ἐστίν,
κοινὴ προσκεῖσθω ἡ ὑπὸ ΕΒΔ·
αἱ ἄρα ὑπὸ ΓΒΕ, ΕΒΔ
τρισοὶ ταῖς ὑπὸ ΓΒΑ, ΑΒΕ, ΕΒΔ
ἴσαι εἰσίν.
πάλιν,
ἐπεὶ ἡ ὑπὸ ΔΒΑ
δυοὶ ταῖς ὑπὸ ΔΒΕ, ΕΒΑ ἴση ἐστίν,
κοινὴ προσκεῖσθω ἡ ὑπὸ ΑΒΓ·
αἱ ἄρα ὑπὸ ΔΒΑ, ΑΒΓ
τρισοὶ ταῖς ὑπὸ ΔΒΕ, ΕΒΑ, ΑΒΓ
ἴσαι εἰσίν.
ἐδείχθησαν δὲ καὶ αἱ ὑπὸ ΓΒΕ, ΕΒΔ
τρισοὶ ταῖς αὐταῖς ἴσαι·
τὰ δὲ τῶ αὐτῶ ἴσα
καὶ ἀλλήλοισ ἐστὶν ἴσα·
καὶ αἱ ὑπὸ ΓΒΕ, ΕΒΔ ἄρα
ταῖς ὑπὸ ΔΒΑ, ΑΒΓ ἴσαι εἰσίν·
ἀλλὰ αἱ ὑπὸ ΓΒΕ, ΕΒΔ
δύο ὀρθαὶ εἰσίν·
καὶ αἱ ὑπὸ ΔΒΑ, ΑΒΓ ἄρα
δυοὶ ὀρθαῖς ἴσαι εἰσίν.

Ἐὰν ἄρα εὐθεῖα
ἐπ' εὐθείαν σταθεῖσα
γωνίας ποιῆ,
ἤτοι δύο ὀρθὰς
ἢ δυοὶν ὀρθαῖς ἴσας
ποιήσῃ [τήρημ].
ὅπερ ἔδει δεῖξαι.



1.14

If to some STRAIGHT,
and at the same point,
two STRAIGHTS,
not lying to the same parts,
the adjacent angles
to two RIGHTS
make equal,
on a STRAIGHT
will be with one another
the STRAIGHTS.

Ἐὰν πρὸς τινὶ εὐθείᾳ
καὶ τῶ πρὸς αὐτῇ σημείῳ
δύο εὐθεῖαι
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δυοὶν ὀρθαῖς ἴσας
ποιῶσιν,
ἐπ' εὐθείας
ἔσσονται ἀλλήλαις
αἱ εὐθεῖαι.

kabul edelim ki çizilmiş olsun,
B noktasından,
ΓΔ doğrusuna,
dik açılarda,
BE.

Dolayısıyla ΓBE ve EBD iki diktir;
ve olduğundan ΓBE
eşit ΓBA ve ABE ikilisine,
EBD her birine eklenmiş olsun.
Dolayısıyla ΓBE ve EBD
eşittirler,
ΓBA, ABE ve EBD üçlüsüne.

Dahası,
olduğundan ΔBA
eşit, ΔBE ve EBA ikilisine,
ABΓ her birine eklenmiş olsun;
dolayısıyla ΔBA ve ABΓ
eşittirler,
ΔBE, EBA ve ABΓ üçlüsüne.

Ve ΓBE ve EBD açıların gösteril-
mişti
eşitliği aynı üçlüye.
Ve aynı şeye eşit olanlar birbirine eşit-
tir;
ve, dolayısıyla, ΓBE ve EBD
eşittirler ΔBA ve ABΓ açılarna;
ama ΓBE ve EBD iki diktir;
ve dolayısıyla ΔBA ve ABΓ
iki dike eşittirler.

Eğer, dolayısıyla, bir doğru,
dikilmiş bir doğrunun üzerine,
yaparsa açılar,
ya iki dik
ya da iki dike eşit
olacak (onları).
—gösterilmesi gereken tam buydu.

Eğer bir doğruya,
ve aynı noktasında,
iki doğru,
aynı tarafında kalmayan,
bitişik açıları,
yaparsa
iki dik açığa eşit
bir doğrudaki
olacaklar birbirleriyle,
doğrular.

For, to some STRAIGHT, AB,
and at the same point, B,
two STRAIGHTS BΓ and BΔ,
not lying to the same parts,
the adjacent angles
ABΓ and ABΔ
equal to two RIGHTS
—suppose they make.

I say that
on a STRAIGHT
with ΓB is BΔ.

For, if it is not
with BΓ on a STRAIGHT,
[namely] BΔ,
let there be,
with BΓ in a STRAIGHT,
BE.

For, since the STRAIGHT AB
has stood¹ to the STRAIGHT ΓBE,
therefore angles ABΓ and ABE
are equal to two RIGHTS.
Also ABΓ and ABΔ
are equal to two RIGHTS.
Therefore ΓBA and ABE
are equal to ΓBA and ABΔ.
In common
suppose there has been taken away
ΓBA;
therefore the remainder ABE
to the remainder ABΔ is equal,
the less to the greater;
which is impossible.
Therefore it is not [the case that]
BE is on a STRAIGHT with ΓB.
Similarly we² shall show that
no other [is so], except BΔ.
Therefore on a STRAIGHT
is ΓB with BΔ.

If, therefore, to some STRAIGHT,
and at the same point,
two STRAIGHTS,
not lying in the same parts,
adjacent angles
two right angles
make,
on a STRAIGHT
will be with one another
the STRAIGHTS;
—just what it was necessary to show.

Πρὸς γὰρ τινὶ εὐθείᾳ τῇ AB
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ B
δύο εὐθεῖαι αἱ BΓ, BΔ
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
τὰς ὑπὸ ABΓ, ABΔ
δύο ὀρθαῖς ἴσας
ποιεῖτωσαν·

λέγω, ὅτι
ἐπ' εὐθείας
ἐστὶ τῇ ΓB ἢ BΔ.

Εἰ γὰρ μὴ ἐστὶ
τῇ BΓ ἐπ' εὐθείας
ἢ BΔ,
ἔστω
τῇ ΓB ἐπ' εὐθείας
ἢ BE.

Ἐπεὶ οὖν εὐθεῖα ἡ AB
ἐπ' εὐθεῖαν τὴν ΓBE ἐφέστηκεν,
αἱ ἄρα ὑπὸ ABΓ, ABE γωνία
δύο ὀρθαῖς ἴσαι εἰσὶν·
εἰσὶ δὲ καὶ αἱ ὑπὸ ABΓ, ABΔ
δύο ὀρθαῖς ἴσαι·
αἱ ἄρα ὑπὸ ΓBA, ABE
ταῖς ὑπὸ ΓBA, ABΔ ἴσαι εἰσὶν.
κοινὴ
ἀφηρήσθω
ἡ ὑπὸ ΓBA·
λοιπὴ ἄρα ἡ ὑπὸ ABE
λοιπῇ τῇ ὑπὸ ABΔ ἐστὶν ἴση,
ἡ ἐλάσσων τῇ μείζονι·
ὅπερ ἐστὶν ἀδύνατον.
οὐκ ἄρα
ἐπ' εὐθείας ἐστὶν ἡ BE τῇ ΓB.
ὁμοίως δὲ δεῖξομεν, ὅτι
οὐδὲ ἄλλη τις πλὴν τῆς BΔ·
ἐπ' εὐθείας ἄρα
ἐστὶν ἡ ΓB τῇ BΔ.

Ἐὰν ἄρα πρὸς τινὶ εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ
δύο εὐθεῖαι
μὴ ἐπὶ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δυσὶν ὀρθαῖς ἴσας
ποιῶσιν,
ἐπ' εὐθείας
ἔσονται ἀλλήλαις
αἱ εὐθεῖαι·
ὅπερ εἶδει δεῖξαι.

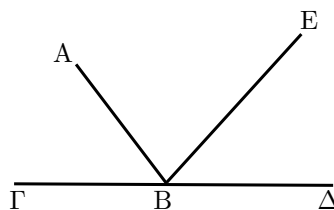
Bir AB doğrusuna,
ve bir B noktasında,
aynı tarafında kalmayan,
iki BΓ ve BΔ doğrularının,
ABΓ ve ABΔ
bitişik açılarının iki dik açı
—olduğu kabul edilsin.

İddia ediyorum ki
BΔ ile ΓB bir doğrudadır.

Çünkü, eğer değilse
bir doğrudadır BΓ ile,
BΔ,
olsun,
bir doğrudadır BΓ ile,
BE.

Çünkü, AB doğrusu
dikiltilmiş olur ΓBE doğrusuna,
dolayısıyla ABΓ ve ABE açıları
eşittirler iki dik açıya.
Ayrıca ABΓ ve ABΔ
eşittirler iki dik açıya.
Dolayısıyla ΓBA ve ABE
eşittirler ΓBA ve ABΔ açılarına.
Ortak ΓBA açısının çıkartıldığı kabul
edilsin.
Dolayısıyla ABE kalanı
eşittir ABΔ kalanına,
küçük olan büyüğe;
ki bu imkansızdır.
Dolayısıyla değildir [durum] şöyle;
BE bir doğrudadır ΓB doğrusuyla.
Benzer şekilde göstereceğiz ki
hiçbiri [öyledir], BΔ dışında.
Dolayısıyla ΓB bir doğrudadır BΔ ile.

Eğer, dolayısıyla, bir doğruya,
ve aynı noktasında,
iki doğru,
aynı tarafında kalmayan,
bitişik açıları,
yaparsa
iki dik açıya eşit
bir doğrudadır
olacaklar birbirleriyle,
doğrular. —gösterilmesi gereken tam
buydu.



¹The English perfect sounds strange here, but the point may be that the standing has already come to be and will continue.

²This seems to be the first use of the first person *plural*.

1.15

If two STRAIGHTS cut one another,
the vertical¹ angles
they make equal to one another.

For, let the STRAIGHTS AB and ΓΔ
cut one another
at the point E.

I say that
equal are
angle AEG to ΔEB,
and ΓEB to AED.

For, since the STRAIGHT AE
has stood to the STRAIGHT ΓΔ,
making angles ΓEA and AED,
therefore angles ΓEA and AED
are equal to two RIGHTS.

Moreover,
since the STRAIGHT ΔE
has stood to the STRAIGHT AB,
making angles AED and ΔEB,
therefore angles AED and ΔEB
are equal to two RIGHTS.

And ΓEA and AED were shown
equal to two RIGHTS;
therefore ΓEA and AED
are equal to AED and ΔEB.

In common
suppose there has been taken away
AED;

therefore the remainder ΓEA
is equal to the remainder ΔEB;
similarly it will be shown that
also ΓEB and ΔEA are equal.²

If, therefore,
two STRAIGHTS cut one another,
the vertical angles
they make equal to one another;
—just what it was necessary to show.

Ἐὰν δύο εὐθεῖαι τέμνωσιν ἀλλήλας,
τὰς κατὰ κορυφὴν γωνίας
ἴσας ἀλλήλαις ποιούσιν.

Δύο γὰρ εὐθεῖαι αἱ AB, ΓΔ
τεμνέτωσαν ἀλλήλας
κατὰ τὸ E σημεῖον·

λέγω, ὅτι
ἴση ἐστὶν
ἡ μὲν ὑπὸ AEG γωνία τῇ ὑπὸ ΔEB,
ἡ δὲ ὑπὸ ΓEB τῇ ὑπὸ AED.

Ἐπεὶ γὰρ εὐθεῖα ἡ AE
ἐπ' εὐθεῖαν τὴν ΓΔ ἐφέστηκε
γωνίας ποιούσα τὰς ὑπὸ ΓEA, AED,
αἱ ἄρα ὑπὸ ΓEA, AED γωνία
δυσὶν ὀρθαῖς ἴσαι εἰσίν.

πάλιν,
ἐπεὶ εὐθεῖα ἡ ΔE
ἐπ' εὐθεῖαν τὴν AB ἐφέστηκε
γωνίας ποιούσα τὰς ὑπὸ AED, ΔEB,
αἱ ἄρα ὑπὸ AED, ΔEB γωνία
δυσὶν ὀρθαῖς ἴσαι εἰσίν.

ἔδειχθησαν δὲ καὶ αἱ ὑπὸ ΓEA, AED
δυσὶν ὀρθαῖς ἴσαι·

αἱ ἄρα ὑπὸ ΓEA, AED
ταῖς ὑπὸ AED, ΔEB ἴσαι εἰσίν.

κοινῇ

ἀφῆρησθῶ
ἡ ὑπὸ AED·

λοιπὴ ἄρα ἡ ὑπὸ ΓEA
λοιπῇ τῇ ὑπὸ ΔEB ἴση ἐστίν·
ὁμοίως δὲ δεῖχθήσεται, ὅτι
καὶ αἱ ὑπὸ ΓEB, ΔEA ἴσαι εἰσίν.

Ἐὰν ἄρα
δύο εὐθεῖαι τέμνωσιν ἀλλήλας,
τὰς κατὰ κορυφὴν γωνίας
ἴσας ἀλλήλαις ποιούσιν·
ὅπερ ἔδει δεῖξαι.

Eğer iki doğru keserse birbirini,
ters açılar
oluşturlar eşit bir birine.

Çünkü, AB ve ΓΔ doğruları
kessinler birbirlerini
E noktasında.

İddia ediyorum ki
eşittirler
AEG açısı ΔEB açısına,
ve ΓEB açısı AED açısına.

Çünkü, AE doğrusu
yerleşmişti ΓΔ doğrusuna,
oluşturur ΓEA ve AED açılarını,
dolayısıyla ΓEA ve AED açıları
eşittirler iki dik açıya.

Dahası,
ΔE doğrusu
dikilmişti AB doğrusuna,
oluşturarak AED ve ΔEB açılarını,
dolayısıyla AED ve ΔEB açıları
eşittirler iki dik açıya.

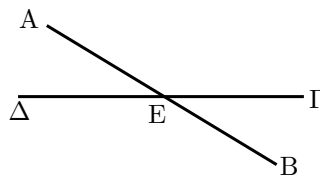
Ve ΓEA ve AED açıların göster-
ilmişti

eşitliği iki dik açıya,
dolayısıyla ΓEA ve AED
eşittirler AED ve ΔEB açılarına.

Ortak AED açısının çıkartılmış
olduğu kabul edilsin;

dolayısıyla ΓEA kalanı
eşittir ΔEB kalanına;
benzer şekilde gösterilecek ki
ΓEB açısı da eşittir ΔEA açısına.

Eğer, dolayısıyla,
iki doğru keserse bir birini,
ters açılar
oluşturlar eşit birbirine
—gösterilmesi gereken tam buydu.



1.16

One of the sides of any triangle
being extended,
the exterior angle
than either

Παντὸς τριγώνου μιᾶς τῶν πλευρῶν
προσεκβληθείσης
ἡ ἔκτος γωνία
ἐκατέρας

Herhangi bir üçgenin kenarlarından
biri
uzatılıldığında,
dış açı

¹The Greek is κατὰ κορυφὴν, which might be translated as 'at a head', just as, in the conclusion of I.10, AB has been cut in two 'at Δ', κατὰ τὸ Δ. But κορυφή and the Latin *vertex* can both mean *crown of the head*, and in anatomical use, the English *vertical* refers

to this crown. Apollonius uses κορυφή for the vertex of a cone [17, pp. 286–7].

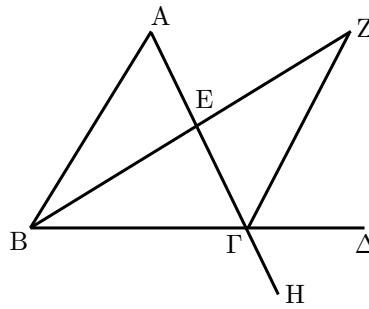
²This is a rare moment when two things are said to be equal *simply*, and not equal to one another.

of the interior and opposite angles is greater.	τῶν ἐντὸς καὶ ἀπεναντίον γωνιῶν μείζων ἐστίν.	her bir iç ve karşıt açıdan büyüktür.
Let there be a triangle, ABΓ, and let there have been extended its side BΓ, to Δ.	Ἐστω τρίγωνον τὸ ABΓ, καὶ προσεκβεβλήσθω αὐτοῦ μία πλευρὰ ἢ BΓ ἐπὶ τὸ Δ.	Olsun, bir ABΓ üçgeni ve uzatılmış olsun onun BΓ kenarı Δ noktasına.
I say that the exterior angle AΓΔ is greater than either of the two interior and opposite angles, ΓBA and BAΓ.	λέγω, ὅτι ἢ ἐκτὸς γωνία ἢ ὑπὸ AΓΔ μείζων ἐστὶν ἑκατέρας τῶν ἐντὸς καὶ ἀπεναντίον τῶν ὑπὸ ΓBA, BAΓ γωνιῶν.	İddia ediyorum ki AΓΔ dış açısı büyüktür her iki ΓBA ve BAΓ iç ve karşıt açılarından.
Suppose AΓ has been cut in two at E, and BE, being joined, —suppose it has been extended on a STRAIGHT to Z, and there has been laid down, equal to BE, EZ, and there has been joined ZΓ, and there has been drawn through AΓ to H.	Τετμησθὼ ἢ AΓ δίχα κατὰ τὸ E, καὶ ἐπιζευχθεῖσα ἢ BE ἐκβεβλήσθω ἐπ' εὐθείας ἐπὶ τὸ Z, καὶ κείσθω τῇ BE ἴση ἢ EZ, καὶ ἐπεζεύχθω ἢ ZΓ, καὶ διήχθω ἢ AΓ ἐπὶ τὸ H.	AΓ kenarı, E noktasından ikiye kesilmiş olsun, ve birleştirilen BE, —uzatılmış olsun Z noktasına bir doğruya ve yerleştirilmiş olsun, BE doğrusuna eşit olan EZ, ve birleştirilmiş olsun ZΓ, ve çizilmiş olsun AΓ doğrusu H noktasına kadar.
Since equal are AE to EΓ, and BE to EZ, the two, AE and EB to the two, EΓ and EZ, are equal, either to either; and angle AEB is equal to angle ZEG; for they are vertical; therefore the base AB is equal to the base ZΓ, and triangle ABE is equal to triangle ZEG, and the remaining angles are equal to the remaining angles, either to either, which the equal sides subtend. Therefore equal are BAE and EΓZ. but greater is EΓΔ than EΓZ; therefore greater [is] AΓΔ than BAE. Similarly BΓ having been cut in two, it will be shown that BΓH, which is AΓΔ, [is] greater than ABΓ.	Ἐπεὶ οὖν ἴση ἐστὶν ἢ μὲν AE τῇ EΓ, ἢ δὲ BE τῇ EZ, δύο δὲ αἱ AE, EB δυοὶ ταῖς EΓ, EZ ἴσαι εἰσὶν ἑκατέρα ἑκατέρᾳ καὶ γωνία ἢ ὑπὸ AEB γωνία τῇ ὑπὸ ZEG ἴση ἐστίν· κατὰ κορυφὴν γάρ· βάσις ἄρα ἢ AB βάσει τῇ ZΓ ἴση ἐστίν, καὶ τὸ ABE τρίγωνον τῷ ZEG τριγώνῳ ἐστὶν ἴσον, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἴσαι εἰσὶν ἑκατέρα ἑκατέρᾳ, ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν· ἴση ἄρα ἐστὶν ἢ ὑπὸ BAE τῇ ὑπὸ EΓZ. μείζων δὲ ἐστὶν ἢ ὑπὸ EΓΔ τῆς ὑπὸ EΓZ· μείζων ἄρα ἢ ὑπὸ AΓΔ τῆς ὑπὸ BAE. Ὅμοίως δὲ τῆς BΓ τετμημένης δίχα δειχθήσεται καὶ ἢ ὑπὸ BΓH, τουτέστιν ἢ ὑπὸ AΓΔ, μείζων καὶ τῆς ὑπὸ ABΓ.	Eşit olduğundan AE, EΓ doğrusuna, ve BE, EZ doğrusuna, AE ve EB ikilisi, eşittirler EΓ ve EZ ikilisinin, her biri birine; ve AEB açısı eşittir ZEG açısına; dikey olduklarından; dolayısıyla AB tabanı eşittir ZΓ tabanına, ve ABE üçgeni eşittir ZEG üçgenine, ve kalan açılar eşittirler kalan açılardan, her biri birine, (yani) eşit kenarları görenler. Dolayısıyla eşittirler EΓΔ ve EΓZ. Ama büyüktür BAE, EΓZ açısından; dolayısıyla büyüktür AΓΔ, BAE açısından. Benzer şekilde ikiye kesilmiş olduğundan BΓ, gösterilecek ki BΓH, AΓΔ açısına eşit olan, büyüktür ABΓ açısından.
Therefore, of any triangle, one of the sides being extended, the exterior angle than either	Παντὸς ἄρα τριγώνου μιᾶς τῶν πλευρῶν προσεκβληθείσης ἢ ἐκτὸς γωνία ἑκατέρας	Dolayısıyla, herhangi bir üçgenin, kenarlarından biri uzatıldığında, dış açı her bir

of the interior and opposite angles
is greater;
—just what it was necessary to show.

τῶν ἐντὸς καὶ ἀπεναντίον γωνιῶν
μείζων ἐστίν·
ὅπερ ἔδει δεῖξαι.

iç ve karşıt açıdan
büyüktür;
—gösterilmesi gereken tam buydu.



1.17

Two angles of any triangle
are less than two RIGHTS
—taken anyhow.

Let there be
a triangle, ABΓ.

I say that
two angles of triangle ABΓ
are less than two RIGHTS
—taken anyhow.

For, suppose there has been extended
BΓ to Δ.

And since, of triangle ABΓ,
AΓΔ is an exterior angle,
it is greater
than the interior and opposite ABΓ.
Let AΓB be added in common;
therefore AΓΔ and AΓB
are greater than ABΓ and BΓA.
But AΓΔ and AΓB
are equal to two RIGHTS;
therefore ABΓ and BΓA
are less than two RIGHTS.
Similarly we shall show that
also BAΓ and AΓB
are less than two RIGHTS,
and yet [so are] ΓAB and ABΓ.

Therefore two angles of any triangle
are greater than two RIGHTS
—taken anyhow;
—just what it was necessary to show.

Παντὸς τριγώνου αἱ δύο γωνίαι
δύο ὀρθῶν ἐλάσσονές εἰσι
πάντῃ μεταλαμβανόμεναι.

Ἐστω
τρίγωνον τὸ ABΓ.

λέγω, ὅτι
τοῦ ABΓ τριγώνου αἱ δύο γωνίαι
δύο ὀρθῶν ἐλάττονές εἰσι
πάντῃ μεταλαμβανόμεναι.

Ἐκβεβλήσθω γὰρ
ἡ BΓ ἐπὶ τὸ Δ.

Καὶ ἐπεὶ τριγώνου τοῦ ABΓ
ἐκτὸς ἐστὶ γωνία ἡ ὑπὸ AΓΔ,
μείζων ἐστὶ
τῆς ἐντὸς καὶ ἀπεναντίον τῆς ὑπὸ ABΓ.
κοινὴ προσκεῖσθω ἡ ὑπὸ AΓB·
αἱ ἄρα ὑπὸ AΓΔ, AΓB
τῶν ὑπὸ ABΓ, BΓA μείζονές εἰσιν.
ἀλλ' αἱ ὑπὸ AΓΔ, AΓB
δύο ὀρθαῖς ἴσαι εἰσὶν·
αἱ ἄρα ὑπὸ ABΓ, BΓA
δύο ὀρθῶν ἐλάσσονές εἰσιν.
ὁμοίως δὴ δεῖξομεν, ὅτι
καὶ αἱ ὑπὸ BAΓ, AΓB
δύο ὀρθῶν ἐλάσσονές εἰσι
καὶ ἔτι αἱ ὑπὸ ΓAB, ABΓ.

Παντὸς ἄρα τριγώνου αἱ δύο γωνίαι
δύο ὀρθῶν ἐλάσσονές εἰσι
πάντῃ μεταλαμβανόμεναι·
ὅπερ ἔδει δεῖξαι.

Herhangi bir üçgenin iki açısı
küçüktür iki dik açıdan
—nasıl alınırsa alınan.

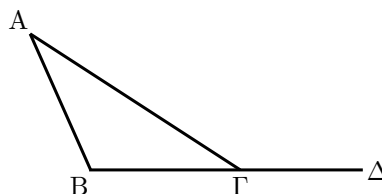
Olsun
bir ABΓ üçgeni.

İddia ediyorum ki
ABΓ üçgeninin iki açısı
küçüktür iki dik açıdan
—nasıl alınırsa alınsın.

Çünkü, uzatılmış olsun,
BΓ, Δ noktasına.

Ve ABΓ üçgeninin,
bir dış açısı olduğundan AΓΔ,
büyüktür
iç ve karşıt ABΓ açısından.
AΓB ortak açısı eklenmiş olsun;
dolayısıyla AΓΔ ve AΓB
büyüktürler ABΓ ve BΓA açılarından.
Ama AΓΔ ve AΓB
eşittirler iki dik açıya;
dolayısıyla ABΓ ve BΓA
küçüktürler iki dik açıdan.
Benzer şekilde göstereceğiz ki
BAΓ ve AΓB de
küçüktürler iki dik açıdan,
ve sonra [öyledirler] ΓAB ve ABΓ.

Dolayısıyla herhangi bir üçgenin iki
açısı
küçüktür iki dik açıdan
—nasıl alınırsa alınsın;
—gösterilmesi gereken tam buydu.



1.18

Of any triangle,
the greater side
subtends the greater angle.¹

For, let there be
a triangle, $AB\Gamma$,
having side $A\Gamma$ greater than AB .

I say that
also angle $AB\Gamma$
is greater than $B\Gamma A$.

For, since $A\Gamma$ is greater than AB ,
suppose there has been laid down,
equal to AB ,
 $A\Delta$,
and let $B\Delta$ be joined.

Since also, of triangle $B\Gamma\Delta$,
angle $A\Delta B$ is exterior,
it is greater
than the interior and opposite $\Delta\Gamma B$;
and $A\Delta B$ is equal to $AB\Delta$,
since side AB is equal to $A\Delta$;
greater therefore
is $AB\Delta$ than $A\Gamma B$;
by much, therefore,
 $AB\Gamma$ is greater
than $A\Gamma B$.

Therefore, of any triangle,
the greater side
subtends the greater angle;
—just what it was necessary to show.

Παντός τριγώνου
ἡ μείζων πλευρὰ
τὴν μείζονα γωνίαν ὑποτείνει.

Ἐστω γὰρ
τρίγωνον τὸ $AB\Gamma$
μείζονα ἔχον τὴν $A\Gamma$ πλευρὰν τῆς AB .

λέγω, ὅτι
καὶ γωνία ἡ ὑπὸ $AB\Gamma$
μείζων ἐστὶ τῆς ὑπὸ $B\Gamma A$.

Ἐπεὶ γὰρ μείζων ἐστὶν ἡ $A\Gamma$ τῆς AB ,
κείσθω
τῇ AB ἴση
ἡ $A\Delta$,
καὶ ἐπεζεύχθω ἡ $B\Delta$.

Καὶ ἐπεὶ τριγώνου τοῦ $B\Gamma\Delta$
ἐκτός ἐστὶ γωνία ἡ ὑπὸ $A\Delta B$,
μείζων ἐστὶ
τῆς ἐντός καὶ ἀπεναντίον τῆς ὑπὸ $\Delta\Gamma B$.
ἴση δὲ ἡ ὑπὸ $A\Delta B$ τῇ ὑπὸ $AB\Delta$,
ἐπεὶ καὶ πλευρὰ ἡ AB τῇ $A\Delta$ ἐστὶν ἴση.
μείζων ἄρα
καὶ ἡ ὑπὸ $AB\Delta$ τῆς ὑπὸ $A\Gamma B$.
πολλῶ ἄρα
ἡ ὑπὸ $AB\Gamma$ μείζων ἐστὶ
τῆς ὑπὸ $A\Gamma B$.

Παντός ἄρα τριγώνου
ἡ μείζων πλευρὰ
τὴν μείζονα γωνίαν ὑποτείνει.
ὅπερ ἔδει δεῖξαι.

Herhangi bir üçgende
daha büyük bir kenar,
daha büyük bir açıyı karşılar.

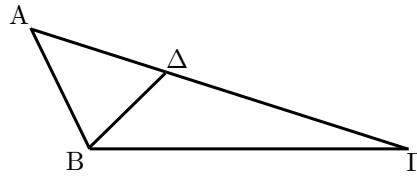
Çünkü, verilmiş olsun
bir $AB\Gamma$ üçgeni,
 $A\Gamma$ kenarı daha büyük olan, AB ke-
narından.

İddia ediyorum ki
 $AB\Gamma$ açısı da
daha büyüktür, $B\Gamma A$ açısından.

Çünkü $A\Gamma$, AB kenarından daha
büyük olduğundan,
yerleştirilmiş olsun,
eşit olan AB kenarına,
 $A\Delta$,
ve $B\Delta$ birleştirilmiş olsun.

Ayrıca, $B\Gamma\Delta$ üçgeninin,
 $A\Delta B$ açısı dış açı olduğundan,
büyüktür
iç ve karşıt $\Delta\Gamma B$ açısından;
ve $A\Delta B$ eşittir $AB\Delta$ açısına,
 AB kenarı eşit olduğundan $A\Delta$ ke-
narına;
büyüktür dolayısıyla
 $AB\Delta$, $A\Gamma B$ açısından;
dolayısıyla, çok daha
büyüktür $AB\Gamma$,
 $A\Gamma B$ açısından.

Dolayısıyla, herhangi bir üçgende
daha büyük bir kenar,
daha büyük bir açıyı karşılar;
—gösterilmesi gereken tam buydu.



1.19

Of any triangle,
under the greater angle
the greater side subtends.¹

For, let there be
a triangle, $AB\Gamma$,
having angle $AB\Gamma$ greater
than $B\Gamma A$.

Παντός τριγώνου
ὑπὸ τὴν μείζονα
γωνίαν ἡ μείζων πλευρὰ ὑποτείνει.

Ἐστω
τρίγωνον τὸ $AB\Gamma$
μείζονα ἔχον τὴν ὑπὸ $AB\Gamma$ γωνίαν
τῆς ὑπὸ $B\Gamma A$.

Herhangi bir üçgende,
daha büyük bir açı,
daha büyük bir kenarca karşılanır.

Çünkü, verilmiş olsun
bir $AB\Gamma$ üçgeni,
 $AB\Gamma$ açısı daha büyük olan,
 $B\Gamma A$ açısından.

¹This enunciation has almost the same words as that of the next proposition. The object of the verb ὑποτείνει is preceded by the preposition ὑπό in the next enunciation, and not here. But the more important difference would seem to be word order: SUBJECT-OBJECT-

VERB here, and OBJECT-SUBJECT-VERB in I.19. This difference in order ensures that I.19 is the converse of I.18.

¹Heath here uses the expedient of the passive: ‘The greater angle is subtended by the greater side.’

I say that
also side AG
is greater than side AB .

For if not,
either AG is equal to AB
or less;
[but] AG is not equal to AB ;
for [if it were],
also ABG would be² equal to AGB ;
but it is not;
therefore AG is not equal to AB .
Nor is AG less than AB ;
for [if it were],
also angle ABG would be [less]
than AGB ;
but it is not;
therefore AG is not less than AB .
And it was shown that
it is not equal.
Therefore AG is greater than AB .

Therefore, of any triangle,
under the greater angle
the greater side subtends;
—just what it was necessary to show.

λέγω, ὅτι
καὶ πλευρὰ ἢ AG
πλευρᾶς τῆς AB μείζων ἐστίν.

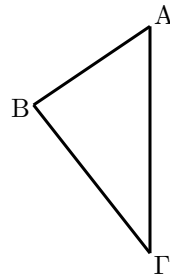
Εἰ γὰρ μή,
ἤτοι ἴση ἐστὶν ἢ AG τῆς AB
ἢ ἐλάσσων·
ἴση μὲν οὖν οὐκ ἔστιν ἢ AG τῆς AB ·
ἴση γὰρ ἂν
ἦν καὶ γωνία ἢ ὑπὸ ABG τῆς ὑπὸ AGB ·
οὐκ ἔστι δέ·
οὐκ ἄρα ἴση ἐστὶν ἢ AG τῆς AB .
οὐδὲ μὴν ἐλάσσων ἐστὶν ἢ AG τῆς AB ·
ἐλάσσων γὰρ
ἂν ἦν καὶ γωνία ἢ ὑπὸ ABG
τῆς ὑπὸ AGB ·
οὐκ ἔστι δέ·
οὐκ ἄρα ἐλάσσων ἐστὶν ἢ AG τῆς AB .
ἐδείχθη δέ, ὅτι
οὐδὲ ἴση ἐστίν.
μείζων ἄρα ἐστὶν ἢ AG τῆς AB .

Παντὸς ἄρα τριγώνου
ὑπὸ τὴν μείζονα γωνίαν
ἢ μείζων πλευρὰ ὑποτείνει·
ὅπερ ἔδει δεῖξαι.

İddia ediyorum ki
 AG kenarı da
daha büyüktür AB kenarından.

Çünkü değil ise,
ya AG eşittir AB kenarına
ya da daha küçüktür;
(ama) AG eşit değildir AB kenarına;
çünkü (eğer olsaydı),
 ABG da eşit olurdu AGB açısına;
ama değildir;
dolayısıyla AG eşit değildir AB ke-
narına.
 AG küçük de değildir AB kenarından;
çünkü (eğer olsaydı),
 ABG açısı da olurdu (küçük)
 AGB açısından;
ama değildir;
dolayısıyla AG küçük değildir AB ke-
narından.
Ve gösterilmişti ki
eşit değildir.
Dolayısıyla AG daha büyüktür AB ke-
narından.

Dolayısıyla, herhangi bir üçgende,
daha büyük bir açı,
daha büyük bir kenarca karşılanır;
—gösterilmesi gereken tam buydu.



1.20

Two sides of any triangle
are greater than the remaining one
—taken anyhow.

For, let there be
a triangle, ABG .

I say that
two sides of triangle ABG
are greater than the remaining one,
—taken anyhow,
 BA and AG , than BG ,
 AB and BG , than AG ,
 BG and GA , than AB .

For, suppose has been drawn through
 BA to a point Δ ,

Παντὸς τριγώνου αἱ δύο πλευραὶ
τῆς λοιπῆς μείζονές εἰσι
πάντη μεταλαμβανόμεναι.

Ἐστω γὰρ
τρίγωνον τὸ ABG ·

λέγω, ὅτι
τοῦ ABG τριγώνου αἱ δύο πλευραὶ
τῆς λοιπῆς μείζονές εἰσι
πάντη μεταλαμβανόμεναι,
αἱ μὲν BA , AG τῆς BG ,
αἱ δὲ AB , BG τῆς AG ,
αἱ δὲ BG , GA τῆς AB .

Διήχθω γὰρ
ἢ BA ἐπὶ τὸ Δ σημεῖον,

Herhangi bir üçgenin iki kenarı
daha büyüktür geriye kalandan
—nasıl seçilirse seçilsin.

Çünkü verilmiş olsun
bir ABG üçgeni.

İddia ediyorum ki
 ABG üçgeninin iki kenarı
daha büyüktür geriye kalandan
—nasıl seçilirse seçilsin,
 BA ve AG , BG kenarından,
 AB ve BG , AG kenarından,
 BG ve GA , AB kenarından.

Çünkü, çizilmiş olsun
 BA kenarı geçerek bir Δ noktasından,

²Literally 'was'; but this conditional use of *was* is archaic in English.

and there has been laid down
 $\Delta\Delta$ equal to $\Gamma\Lambda$,
 and there has been joined
 $\Delta\Gamma$.

Since ΔA is equal to $A\Gamma$,
 equal also is
 angle $A\Delta\Gamma$ to $A\Gamma\Delta$.
 Therefore $B\Gamma\Delta$ is greater than $A\Delta\Gamma$;
 also, since there is a triangle, $\Delta\Gamma B$,¹
 having angle $\Gamma B\Delta$ greater
 than $\Delta B\Gamma$,
 and under the greater angle
 the greater side subtends,
 therefore ΔB is greater than $B\Gamma$.
 But ΔA is equal to $A\Gamma$;
 therefore BA and $A\Gamma$ are greater
 than $B\Gamma$;
 similarly we shall show that
 AB and $B\Gamma$ than ΓA
 are greater,
 and $B\Gamma$ and ΓA than AB .

Therefore two sides of any triangle
 are greater than the remaining one
 —taken anyhow;
 —just what it was necessary to show.

καὶ κείσθω
 τῆ $\Gamma\Lambda$ ἴση ἡ $\Delta\Delta$,
 καὶ ἐπέζεύχθω
 ἡ $\Delta\Gamma$.

Ἐπεὶ οὖν ἴση ἐστὶν ἡ ΔA τῆ $A\Gamma$,
 ἴση ἐστὶ καὶ
 γωνία ἡ ὑπὸ $A\Delta\Gamma$ τῆ ὑπὸ $A\Gamma\Delta$.
 μείζων ἄρα ἡ ὑπὸ $B\Gamma\Delta$ τῆς ὑπὸ $A\Delta\Gamma$.
 καὶ ἐπεὶ τρίγωνόν ἐστι τὸ $\Delta\Gamma B$
 μείζονα ἔχον τὴν ὑπὸ $B\Gamma\Delta$ γωνίαν
 τῆς ὑπὸ $B\Delta\Gamma$,
 ὑπὸ δὲ τὴν μείζονα γωνίαν
 ἡ μείζων πλευρὰ ὑποτείνει,
 ἡ ΔB ἄρα τῆς $B\Gamma$ ἐστὶ μείζων.
 ἴση δὲ ἡ ΔA τῆ $A\Gamma$.
 μείζονες ἄρα αἱ BA , $A\Gamma$
 τῆς $B\Gamma$.
 ὁμοίως δὲ δεῖξομεν, ὅτι
 καὶ αἱ μὲν AB , $B\Gamma$ τῆς ΓA
 μείζονές εἰσιν,
 αἱ δὲ $B\Gamma$, ΓA τῆς AB .

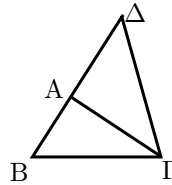
Παντὸς ἄρα τριγώνου αἱ δύο πλευραὶ
 τῆς λοιπῆς μείζονές εἰσι
 πάντῃ μεταλαμβάνομεναι.
 ὅπερ ἔδει δεῖξαι.

ve yerleştirilmiş olsun
 $\Delta\Delta$, $\Gamma\Lambda$ kenarına eşit olan,
 ve birleştirilmiş olsun
 $\Delta\Gamma$.

ΔA eşit olduğundan $A\Gamma$ kenarına,
 eşittir ayrıca
 $A\Delta\Gamma$, $A\Gamma\Delta$ açısına.
 Dolayısıyla $B\Gamma\Delta$ büyüktür, $A\Delta\Gamma$
 açısından;
 yine, $\Delta\Gamma B$, bir üçgen olduğundan,
 $B\Gamma\Delta$ daha büyük olan
 $B\Delta\Gamma$ açısından,
 daha büyük açı
 daha büyük kenarca karşılandığından,
 dolayısıyla ΔB büyüktür $B\Gamma$ kenarın-
 dan.
 Ama ΔA eşittir $A\Gamma$ kenarına;
 dolayısıyla BA ve $A\Gamma$ büyüktürler
 $B\Gamma$ kenarından;
 benzer şekilde göstereceğiz ki
 AB ve $B\Gamma$, ΓA kenarından
 büyüktürler,
 ve $B\Gamma$ ve ΓA , AB kenarından.

Dolayısıyla, herhangi bir üçgenin iki
 kenarı
 daha büyüktür geriye kalandan
 —nasıl seçilirse seçilsin;
 —gösterilmesi gereken tam buydu.

Z



1.21

If, of a triangle,
 on one of the sides,
 from its extremities,
 two STRAIGHTS
 be constructed within,¹
 the constructed [STRAIGHTS],
 than the remaining two sides of the
 triangle
 will be less,
 but will contain the a greater angle.

For, of a triangle, $AB\Gamma$,
 on one of the sides, $B\Gamma$,
 from its extremities, B and Γ ,
 suppose two STRAIGHTS have been
 constructed within,
 $B\Delta$ and $\Delta\Gamma$.

Ἐὰν τριγώνου
 ἐπὶ μιᾶς τῶν πλευρῶν
 ἀπὸ τῶν περάτων
 δύο εὐθεῖαι
 ἐντὸς συσταθῶσιν,¹
 αἱ συσταθεῖσαι
 τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν
 ἐλάττονες μὲν ἔσονται,
 μείζονα δὲ γωνίαν περιέξουσιν.

Τριγώνου γὰρ τοῦ $AB\Gamma$
 ἐπὶ μιᾶς τῶν πλευρῶν τῆς $B\Gamma$
 ἀπὸ τῶν περάτων τῶν B , Γ
 δύο εὐθεῖαι ἐντὸς συνεστάτωσαν
 αἱ $B\Delta$, $\Delta\Gamma$.

Eğer bir üçgende,
 kenarlardan birinin
 uçlarından,
 iki doğru
 içeride inşa edilirse,
 inşa edilen doğrular,
 üçgenin geriye kalan iki kenarından
 daha küçük olacak,
 ama daha büyük bir açıyı içerecekler.

Çünkü, $AB\Gamma$ üçgeninin,
 bir $B\Gamma$ kenarının
 B ve Γ uçlarından,
 içeride iki doğru inşa edilmiş olsun;
 $B\Delta$ ve $\Delta\Gamma$.

¹Heath's version is, 'Since DCB [$\Delta\Gamma B$] is a triangle. . .'

¹Here the Greek verb, $\sigmaυ\sigma\tau\eta\mu\iota$, is the same one used in I.1 for the construction of a triangle on a given straight line. Is it supposed

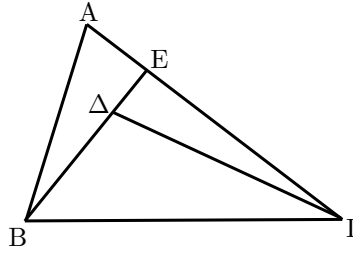
to be obvious to the reader, even *without* a diagram, that now the two constructed straight lines are supposed to meet at a point? See also I.2 and note.

<p>I say that $B\Delta$ and $\Delta\Gamma$ than the remaining two sides of the triangle, BA and $A\Gamma$, are less, but contain a greater angle, $B\Delta\Gamma$, than $BA\Gamma$.</p>	<p>λέγω, ὅτι αἱ $B\Delta$, $\Delta\Gamma$ τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν τῶν BA, $A\Gamma$ ἐλάσσονες μὲν εἰσιν, μεῖζονα δὲ γωνίαν περιέχουσι τὴν ὑπὸ $B\Delta\Gamma$ τῆς ὑπὸ $BA\Gamma$.</p>	<p>İddia ediyorum ki $B\Delta$ ve $\Delta\Gamma$ üçgenin geriye kalan iki BA ve $A\Gamma$ kenarından, daha küçütürler, ama içerirler, $BA\Gamma$ açısından daha büyük $B\Delta\Gamma$ açısını.</p>
<p>For, let $B\Delta$ be drawn through to E.</p>	<p>Διήχθῳ γὰρ ἡ $B\Delta$ ἐπὶ τὸ E.</p>	<p>Çünkü, $B\Delta$ çizilmiş olsun E noktasına doğru.</p>
<p>And since, of any triangle, two sides than the remaining one are greater, of the triangle ABE, the two sides AB and AE are greater than BE; suppose has been added in common EG; therefore BA and $A\Gamma$ than BE and EG are greater. Moreover, since, of the triangle GED, the two sides GE and ED are greater than GD, suppose has been added in common ΔB; therefore GE and EB than GD and ΔB are greater. But than BE and EG BA and $A\Gamma$ were shown greater; therefore by much BA and $A\Gamma$ than $B\Delta$ and $\Delta\Gamma$ are greater.</p>	<p>καὶ ἐπεὶ παντὸς τριγώνου αἱ δύο πλευραὶ τῆς λοιπῆς μεῖζονές εἰσιν, τοῦ ABE ἄρα τριγώνου αἱ δύο πλευραὶ αἱ AB, AE τῆς BE μεῖζονές εἰσιν· κοινὴ προσκεῖσθω ἡ EG· αἱ ἄρα BA, $A\Gamma$ τῶν BE, EG μεῖζονές εἰσιν. πάλιν, ἐπεὶ τοῦ GED τριγώνου αἱ δύο πλευραὶ αἱ GE, ED τῆς GD μεῖζονές εἰσιν, κοινὴ προσκεῖσθω ἡ ΔB· αἱ GE, EB ἄρα τῶν GD, ΔB μεῖζονές εἰσιν. ἀλλὰ τῶν BE, EG μεῖζονες ἐδείχθησαν αἱ BA, $A\Gamma$· πολλῶ ἄρα αἱ BA, $A\Gamma$ τῶν $B\Delta$, $\Delta\Gamma$ μεῖζονές εἰσιν.</p>	<p>Ve herhangi bir üçgenin iki kenarı, geriye kalandan büyük olduğundan, ABE üçgeninin, iki kenarı, AB ve AE büyüktür BE kenarından; ortak olarak eklenmiş olsun EG; dolayısıyla BA ve $A\Gamma$, BE ve EG ke- narlarından büyüktürler. Dahası, GED üçgeninin, iki kenarları, GE ve ED büyüktür GD kenarından, ortak olarak eklenmiş olsun ΔB; dolayısıyla GE ve EB, GD ve ΔB ke- narlarından büyüktürler. Ama BE ve EG kenarlarından BA ve $A\Gamma$ kenarlarının gösterilmişti büyüklüğü; dolayısıyla çok daha büyüktür BA ve $A\Gamma$, $B\Delta$ ve $\Delta\Gamma$ kenarlarından.</p>
<p>Again, since of any triangle the external angle than the interior and opposite angle is greater, therefore, of the triangle $\Gamma\Delta E$ the exterior angle $B\Delta\Gamma$ is greater than $\Gamma E\Delta$. For the same [reason] again, of the triangle ABE, the exterior angle $\Gamma E B$ is greater than $BA\Gamma$. But than $\Gamma E B$ $B\Delta\Gamma$ was shown greater; therefore by much $B\Delta\Gamma$ is greater than $BA\Gamma$.</p>	<p>Πάλιν, ἐπεὶ παντὸς τριγώνου ἡ ἔκτος γωνία τῆς ἐντὸς καὶ ἀπεναντίον μεῖζων ἐστίν, τοῦ $\Gamma\Delta E$ ἄρα τριγώνου ἡ ἔκτος γωνία ἡ ὑπὸ $B\Delta\Gamma$ μεῖζων ἐστὶ τῆς ὑπὸ $\Gamma E\Delta$. διὰ ταῦτὰ τοίνυν καὶ τοῦ ABE τριγώνου ἡ ἔκτος γωνία ἡ ὑπὸ $\Gamma E B$ μεῖζων ἐστὶ τῆς ὑπὸ $BA\Gamma$. ἀλλὰ τῆς ὑπὸ $\Gamma E B$ μεῖζων ἐδείχθη ἡ ὑπὸ $B\Delta\Gamma$· πολλῶ ἄρα ἡ ὑπὸ $B\Delta\Gamma$ μεῖζων ἐστὶ τῆς ὑπὸ $BA\Gamma$.</p>	<p>Tekrar, herhangi bir üçgenin dış açısı iç ve karşıt açıdan daha büyüktür, dolayısıyla, $\Gamma\Delta E$ üçgeninin dış açısı $B\Delta\Gamma$ büyüktür $\Gamma E\Delta$ açısından. Aynı [sebep] tekrar, ABE üçgeninin, dış açısı $\Gamma E B$ büyüktür $BA\Gamma$ açısından. Ama $\Gamma E B$ açısından, $B\Delta\Gamma$ açısının büyüklüğü gösterilmişti; dolayısıyla çok daha büyüktür $B\Delta\Gamma$, $BA\Gamma$ açısından.</p>
<p>If, therefore, of a triangle, on one of the sides, from its extremities, two STRAIGHTS be constructed within, the constructed [STRAIGHTS], than the remaining two sides of the</p>	<p>Ἐὰν ἄρα τριγώνου ἐπὶ μιᾶς τῶν πλευρῶν ἀπὸ τῶν περάτων δύο εὐθεῖαι ἐντὸς συσταθῶσιν, αἱ συσταθεῖσαι τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν</p>	<p>Eğer, dolayısıyla, bir üçgenin, kenarlardan birinin uçlarından, iki doğru içeride inşa edilirse, inşa edilen doğrular, üçgenin geriye kalan iki kenarından</p>

triangle
will be less,
but will contain the a greater angle;
—just what it was necessary to show.

ἐλάττονας μὲν εἰσιν,
μεῖζονα δὲ γωνίαν περιέχουσιν·
ὅπερ ἔδει δεῖξαι.

daha küçük olacak,
ama daha büyük bir açıyı içerecekler;
—gösterilmesi gereken tam buydu.



1.22

From three STRAIGHTS,
which are equal
to three given [STRAIGHTS],
a triangle to be constructed;
and it is necessary
for two than the remaining one
to be greater
[because of any triangle,
two sides
are¹ greater than the remaining one
taken anyhow].

Let be
the given three STRAIGHTS
A, B, and Γ,
of which two than the remaining one
are greater,
taken anyhow,
A and B than Γ,
A and Γ than B,
and B and Γ than A.

Is is necessary
from equals to A, B, and Γ
for a triangle to be constructed.

Suppose there is laid out
some straight line, ΔE,
bounded at Δ,
but unbounded at E,
and there is laid down
ΔZ equal to A,
ZH equal to B,
and HΘ equal to Γ;
and to center Z
at distance ZΔ
a circle has been drawn, ΔΚΛ;
moreover,
to center H,
at distance HΘ,
circle ΚΛΘ has been drawn,

Ἐκ τριῶν εὐθειῶν,
αἱ εἰσιν ἴσαι
τρισὶ ταῖς δοθείσαις [εὐθείαις],
τρίγωνον συστήσασθαι·
δεῖ δὲ²
τὰς δύο τῆς λοιπῆς
μεῖζονας εἶναι
πάντη μεταλαμβανομένας
[διὰ τὸ καὶ παντὸς τριγώνου
τὰς δύο πλευρὰς
τῆς λοιπῆς μεῖζονας εἶναι
πάντη μεταλαμβανομένας].

Ἐστωσαν
αἱ δοθεῖσαι τρεῖς εὐθεῖαι
αἱ A, B, Γ,
ᾧν αἱ δύο τῆς λοιπῆς
μεῖζονες ἔστωσαν
πάντη μεταλαμβανόμεναι,
αἱ μὲν A, B τῆς Γ,
αἱ δὲ A, Γ τῆς B,
καὶ ἔτι αἱ B, Γ τῆς A·

δεῖ δὴ
ἐκ τῶν ἴσων ταῖς A, B, Γ
τρίγωνον συστήσασθαι.

Ἐκχεῖσθω
τις εὐθεῖα ἡ ΔE
πεπερασμένη μὲν κατὰ τὸ Δ
ἄπειρος δὲ κατὰ τὸ E,
καὶ κείσθω
τῆ μὲν A ἴση ἡ ΔZ,
τῆ δὲ B ἴση ἡ ZH,
τῆ δὲ Γ ἴση ἡ HΘ·
καὶ κέντρῳ μὲν τῷ Z,
διαστήματι δὲ τῷ ZΔ
κύκλος γεγράφθω ὁ ΔΚΛ·
πάλιν
κέντρῳ μὲν τῷ H,
διαστήματι δὲ τῷ HΘ
κύκλος γεγράφθω ὁ ΚΛΘ,

Üç doğrudan,
eşit olan
verilmiş üç doğruya,
bir üçgen oluşturulması;
ve gereklidir
ikisinin, kalandan
daha büyük olması
(çünkü herhangi bir üçgenin,
iki kenarı
büyüktür geriye kalandan
nasıl seçilirse seçilsin).

Verilmiş olsun
üç doğru
A, B, ve Γ,
ikisi, kalandan
büyük olan,
nasıl seçilirse seçilsin,
A ile B, Γ kenarından,
A ile Γ, B kenarından,
ve B ile Γ, A kenarından.

Gereklidir
A, B ve Γ doğrularına eşit olanlardan
bir üçgenin inşa edilmesi.

Yerleştirilmiş olsun
bir ΔE doğrusu,
Δ noktasında sınırlanmış,
ama E noktasında sınırlandırılmamış,
yerleştirilmiş olsun
A doğrusuna eşit ΔZ,
B doğrusuna eşit ZH,
ve Γ doğrusuna eşit HΘ ;
ve Z merkezine
ZΔ uzaklığında
bir ΔΚΛ çemberi çizilmiş olsun;
dahası,
H merkezine,
HΘ uzaklığında,
ΚΛΘ çemberi çizilmiş olsun,

¹In the Greek this is the infinitive εἶναι 'to be', as in the previous clause.

²According to Heiberg, the manuscripts have δεῖ δὴ here, as at

the beginnings of specifications (see §); but Proclus and Eutocius have δεῖ δέ in their commentaries.

and KZ and KH have been joined.

I say that
from three STRAIGHTS
equal to A, B, and Γ,
a triangle has been constructed, KZH.

For, since the point Z
is the center of circle ΔΚΛ,
ΖΔ is equal to ΖΚ;
but ΖΔ is equal to Α.
And ΚΖ is therefore equal to Α.
Moreover,
since the point Η
is the center of circle ΑΚΘ,
ΗΘ is equal to ΗΚ;
but ΗΘ is equal to Γ;
and ΚΗ is therefore equal to Γ.
and ΖΗ is equal to Β;
therefore the three STRAIGHTS,
ΚΖ, ΖΗ, and ΗΚ
are equal to the three, Α, Β, and Γ.

Therefore, from the three STRAIGHTS
ΚΖ, ΖΗ, and ΗΚ,
which are equal
to the three given STRAIGHTS
Α, Β, and Γ,
a triangle has been constructed, ΚΖΗ;
—just what it was necessary to show.

Dolayısıyla, üç doğrudan;
ΚΖ, ΖΗ ve ΗΚ,
eşit olan
verilmiş üç doğruya
Α, Β ve Γ,
bir ΚΖΗ üçgeni inşa edilmiştir;
—gösterilmesi gereken tam buydu.

καὶ ἐπεζεύχθησαν αἱ ΚΖ, ΚΗ·

λέγω, ὅτι
ἐκ τριῶν εὐθειῶν
τῶν ἴσων ταῖς Α, Β, Γ
τρίγωνον συνέσταται τὸ ΚΖΗ.

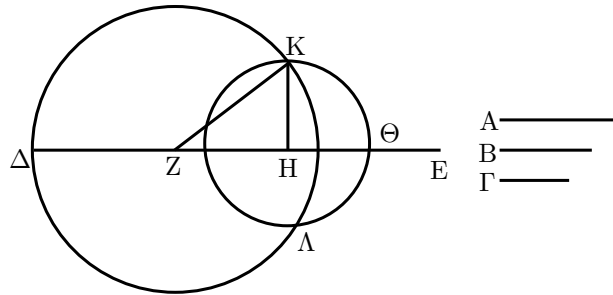
Ἐπεὶ γὰρ τὸ Ζ σημεῖον
κέντρον ἐστὶ τοῦ ΔΚΛ κύκλου,
ἴση ἐστὶν ἡ ΖΔ τῇ ΖΚ·
ἀλλὰ ἡ ΖΔ τῇ Α ἐστὶν ἴση.
καὶ ἡ ΚΖ ἄρα τῇ Α ἐστὶν ἴση.
πάλιν,
ἐπεὶ τὸ Η σημεῖον
κέντρον ἐστὶ τοῦ ΑΚΘ κύκλου,
ἴση ἐστὶν ἡ ΗΘ τῇ ΗΚ·
ἀλλὰ ἡ ΗΘ τῇ Γ ἐστὶν ἴση·
καὶ ἡ ΚΗ ἄρα τῇ Γ ἐστὶν ἴση·
ἐστὶ δὲ καὶ ἡ ΖΗ τῇ Β ἴση·
αἱ τρεῖς ἄρα εὐθεῖαι
αἱ ΚΖ, ΖΗ, ΗΚ
τριῶν ταῖς Α, Β, Γ ἴσαι εἰσίν.

Ἐκ τριῶν ἄρα εὐθειῶν
τῶν ΚΖ, ΖΗ, ΗΚ,
αἱ εἰσὶν ἴσαι
τριῶν ταῖς δοθείσαις εὐθείαις
ταῖς Α, Β, Γ,
τρίγωνον συνέσταται τὸ ΚΖΗ·
ὅπερ ἔδει ποιῆσαι.

ve ΚΖ ile ΚΗ birleştirilmiş olsun.

İddia ediyorum ki
üç doğrudan
Α, Β ve Γ doğrularına eşit olan
bir ΚΖΗ üçgeni inşa edilmiştir.

Çünkü merkezi olduğundan Ζ noktası,
ΔΚΛ çemberinin,
ΖΔ eşittir ΖΚ doğrusuna;
ama ΖΔ eşittir Α doğrusuna.
Ve ΚΖ dolayısıyla Α doğrusuna eşittir.
Dahası,
merkezi olduğundan Η noktası
ΑΚΘ çemberinin,
ΗΘ eşittir ΗΚ doğrusuna;
ama ΗΘ eşittir Γ doğrusuna;
ve ΚΗ dolayısıyla Γ doğrusuna eşittir.
ve ΖΗ eşittir Β doğrusuna;
dolayısıyla üç doğru,
ΚΖ, ΖΗ ve ΗΚ
eşittirler Α, Β ve Γ üçlüsüne.



1.23

At the given STRAIGHT,
and at the given point on it,
equal to the given rectilinear angle,
a rectilinear angle to be constructed.

Let be
the given STRAIGHT ΑΒ,
the point on it, Α,
the given rectilinear angle,
ΔΓΕ.

It is necessary then,

Πρὸς τῇ δοθείσῃ εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ
τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ ἴσην
γωνίαν εὐθύγραμμον συστήσασθαί.

Ἐστω
ἡ μὲν δοθείσα εὐθεῖα ἡ ΑΒ,
τὸ δὲ πρὸς αὐτῇ σημεῖον τὸ Α,
ἡ δὲ δοθείσα γωνία εὐθύγραμμος
ἡ ὑπὸ ΔΓΕ·

δεῖ δὴ

Verilmiş bir doğruya,
ve üzerinde verilmiş noktada,
verilmiş düzkenar açıya eşit olan,
bir düzkenar açı inşa edilmesi.

Verilmiş olsun
ΑΒ doğrusu,
üzerindeki Α noktası,
verilmiş olsun düzkenar açı,
ΔΓΕ.

Gereklidir şimdi,

on the given STRAIGHT, AB,
and at the point A on it,
to the given rectilinear angle
 $\Delta\Gamma\epsilon$
equal,
for a rectilinear angle
to be constructed.

Suppose there have been chosen
on either of $\Gamma\Delta$ and $\Gamma\epsilon$
random points Δ and ϵ ,
and $\Delta\epsilon$ has been joined,
and from three STRAIGHTS,
which are equal to the three,
 $\Gamma\Delta$, $\Delta\epsilon$, and $\Gamma\epsilon$,
triangle AZH has been constructed,
so that equal are
 $\Gamma\Delta$ to AZ,
 $\Gamma\epsilon$ to AH,
and $\Delta\epsilon$ to ZH.

Since then the two, $\Delta\Gamma$ and $\Gamma\epsilon$,
are equal to the two, ZA and AH,
either to either,
and the base $\Delta\epsilon$ to the base ZH
is equal,
therefore the angle $\Delta\Gamma\epsilon$
is equal to ZAH.

Therefore, on the given STRAIGHT,
AB,
and at the point A on it,
equal to the given rectilinear angle,
 $\Delta\Gamma\epsilon$,
the rectilinear angle ZAH has been
constructed;
—just what it was necessary to do.

πρὸς τῇ δοθείσῃ εὐθείᾳ τῇ AB
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A
τῇ δοθείσῃ γωνίᾳ εὐθυγράμμῳ
τῇ ὑπὸ $\Delta\Gamma\epsilon$
ἴσην
γωνίαν εὐθύγραμμον
συστήσασθαι.

Εἰλήφθω
ἐφ' ἑκατέρας τῶν $\Gamma\Delta$, $\Gamma\epsilon$
τυχόντα σημεία τὰ Δ , ϵ ,
καὶ ἐπεζεύχθω ἡ $\Delta\epsilon$.
καὶ ἐκ τριῶν εὐθειῶν,
αἱ εἰσὶν ἴσαι τρισὶ
ταῖς $\Gamma\Delta$, $\Delta\epsilon$, $\Gamma\epsilon$,
τρίγωνον συνεστάτω τὸ AZH,
ὥστε ἴσην εἶναι
τὴν μὲν $\Gamma\Delta$ τῇ AZ,
τὴν δὲ $\Gamma\epsilon$ τῇ AH,
καὶ εἶτι τὴν $\Delta\epsilon$ τῇ ZH.

Ἐπεὶ οὖν δύο αἱ $\Delta\Gamma$, $\Gamma\epsilon$
δύο ταῖς ZA, AH ἴσαι εἰσὶν
ἑκατέρα ἑκατέρᾳ,
καὶ βάσις ἡ $\Delta\epsilon$ βάσει τῇ ZH
ἴση,
γωνία ἄρα ἡ ὑπὸ $\Delta\Gamma\epsilon$ γωνία
τῇ ὑπὸ ZAH ἴσθαι.

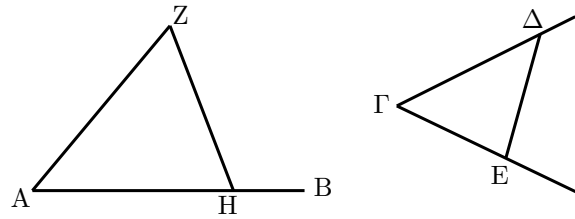
Πρὸς ἄρα τῇ δοθείσῃ εὐθείᾳ
τῇ AB
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A
δοθείσῃ γωνίᾳ εὐθυγράμμῳ τῇ ὑπὸ
 $\Delta\Gamma\epsilon$ ἴσην
γωνίαν εὐθύγραμμον συνέσταιται ἡ ὑπὸ
ZAH.
ὅπερ ἔδει ποιῆσαι.

verilmiş AB doğrusunda,
ve üzerindeki A noktasında,
verilmiş düzkenar
 $\Delta\Gamma\epsilon$ açısına
eşit,
bir düzkenar açının
inşa edilmesi.

Seçilmiş olsun
 $\Gamma\Delta$ ve $\Gamma\epsilon$ doğrularının her birinden
rastgele Δ ve ϵ noktaları,
ve $\Delta\epsilon$ birleştirilmiş olsun,
ve üç doğrudan,
eşit olan verilmiş üç
 $\Gamma\Delta$, $\Delta\epsilon$ ve $\Gamma\epsilon$ doğrularına,
bir AZH üçgen inşa edilmiş olsun
öyle ki, eşit olsun
 $\Gamma\Delta$, AZ doğrusuna,
 $\Gamma\epsilon$, AH doğrusuna, ve $\Delta\epsilon$, ZH
doğrusuna.

O zaman $\Delta\Gamma$ ve $\Gamma\epsilon$ ikilisi,
eşit olduğundan ZA ve AH ikilisinin,
her biri birine,
ve $\Delta\epsilon$ tabanı, ZH tabanına
eşit,
dolayısıyla $\Delta\Gamma\epsilon$ açısı
eşittir ZAH açısına.

Dolayısıyla,
AB doğrusunda,
ve üzerindeki A noktasında,
verilen düzkenar $\Delta\Gamma\epsilon$ açısına eşit,
ZAH düzkenar açısı inşa edilmiştir;
—yapılması gereken tam buydu.



1.24

If two triangles
two sides
to two sides
have equal
either to either,
but angle
than angle
have greater,
[namely] that by the equal sides
contained,
also base
than base
they will have greater.

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχῃ
ἑκατέραν ἑκατέρᾳ,
τὴν δὲ γωνίαν
τῆς γωνίας
μείζονα ἔχῃ
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν
τῆς βάσεως
μείζονα ἔξει.

Eğer iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına
eşitse,
her biri birine,
ama açısı
açısından
büyükse,
[yani] eşit kenarlarca
içerilen(ler),
tabanı da
tabanından
büyük olacak.

Let there be
two triangles, $AB\Gamma$ and ΔEZ ,
—two sides, AB and $A\Gamma$,
to two sides, ΔE and ΔZ ,
having equal,
either to either,
 AB to ΔE ,
and $A\Gamma$ to ΔZ ,
—and the angle at A ,
than the angle at Δ ,
let it be greater.

I say that
also the base $B\Gamma$
than the base EZ
is greater.

For since [it] is greater,
[namely] angle BAG
than angle $E\Delta Z$,
suppose has been constructed
on the STRAIGHT, ΔE ,
and at the point Δ on it,
equal to angle BAG ,
 $E\Delta H$,
and suppose is laid down,
to either of $A\Gamma$ and ΔZ equal,
 ΔH ,
and suppose have been joined
 EH and ZH .

Since [it] is equal,
 AB to ΔE ,
and $A\Gamma$ to ΔH ,
the two, BA and $A\Gamma$,
to the two, $E\Delta$ and ΔH ,
are equal,
either to either;
and angle BAG
to angle $E\Delta H$ is equal;
therefore the base $B\Gamma$
to the base EH is equal.
Moreover,
since [it] is equal,
[namely] ΔZ to ΔH ,
[it] too is equal,
[namely] angle ΔHZ to ΔZH ;
therefore [it] is greater,
[namely] ΔZH than EZH ;
therefore [it] is much greater,
[namely] EZH than EHZ .
And since there is a triangle, EZH ,
having greater
angle EZH than EHZ ,
and the greater angle,
—the greater side subtends it;
greater therefore also is
side EH than EZ .
And [it] is equal, EH to $B\Gamma$;
greater therefore is $B\Gamma$ than EZ .

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , $A\Gamma$
ταῖς δύο πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἑκατέραν ἑκατέρα,
τὴν μὲν AB τῆ ΔE
τὴν δὲ $A\Gamma$ τῆ ΔZ ,
ἢ δὲ πρὸς τῷ A γωνία
τῆς πρὸς τῷ Δ γωνίας
μείζων ἔστω·

λέγω, ὅτι
καὶ βάσις ἢ $B\Gamma$
βάσεως τῆς EZ
μείζων ἔστί·

Ἐπεὶ γὰρ μείζων
ἢ ὑπὸ BAG γωνία
τῆς ὑπὸ $E\Delta Z$ γωνίας,
συνεστάτω
πρὸς τῆ ΔE εὐθείᾳ
καὶ τῷ πρὸς αὐτῆ σημείῳ τῷ Δ
τῆ ὑπὸ BAG γωνία ἴση
ἢ ὑπὸ $E\Delta H$,
καὶ κείσθω
ὁποτέρᾳ τῶν $A\Gamma$, ΔZ ἴση
ἢ ΔH ,
καὶ ἐπεζεύχθωσαν
αἱ EH , ZH .

Ἐπεὶ οὖν ἴση ἔστί·
ἢ μὲν AB τῆ ΔE ,
ἢ δὲ $A\Gamma$ τῆ ΔH ,
δύο δὲ αἱ BA , $A\Gamma$
δυσὶ ταῖς $E\Delta$, ΔH
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρα·
καὶ γωνία ἢ ὑπὸ BAG
γωνία τῆ ὑπὸ $E\Delta H$ ἴση·
βάσις ἄρα ἢ $B\Gamma$
βάσει τῆ EH ἔστί· ἴση.
πάλιν,
ἐπεὶ ἴση ἔστί·
ἢ ΔZ τῆ ΔH ,
ἴση ἔστί· καὶ
ἢ ὑπὸ ΔHZ γωνία τῆ ὑπὸ ΔZH ·
μείζων ἄρα
ἢ ὑπὸ ΔZH τῆς ὑπὸ EZH ·
πολλῶ ἄρα μείζων ἔστί·
ἢ ὑπὸ EZH τῆς ὑπὸ EHZ .
καὶ ἐπεὶ τρίγωνόν ἐστι τὸ EZH
μείζονα ἔχον
τὴν ὑπὸ EZH γωνίαν τῆς ὑπὸ EHZ ,
ὑπὸ δὲ τὴν μείζονα γωνίαν
ἢ μείζων πλευρὰ ὑποτείνει,
μείζων ἄρα καὶ
πλευρὰ ἢ EH τῆς EZ .
ἴση δὲ ἢ EH τῆ $B\Gamma$ ·
μείζων ἄρα καὶ ἢ $B\Gamma$ τῆς EZ .

Verilmiş olsun
iki $AB\Gamma$ ve ΔEZ üçgeni,
— iki AB ve $A\Gamma$ kenarı,
iki ΔE ve ΔZ kenarına,
eşit olan,
her biri birine,
 AB , ΔE kenarına,
ve $A\Gamma$, ΔZ kenarına,
—ve A noktasındaki açısı,
 Δ doktasındakinden,
büyük olsun.

İddia ediyorum ki
 $B\Gamma$ tabanı da
 EZ tabanından
büyüktür.

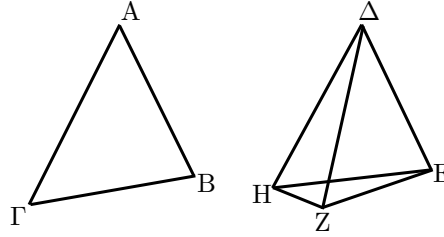
Çünkü büyük olduğundan,
 BAG açısı
 $E\Delta Z$ açısından,
inşa edilmiş olsun
 ΔE doğrusunda,
ve üzerindeki Δ noktasında,
 BAG açısına eşit,
 $E\Delta H$,
ve yerleştirilmiş olsun
 $A\Gamma$ ve ΔZ kenarlarının ikisine de eşit,
 ΔH ,
ve birleştirilmiş olsun
 EH ve ZH .

Eşit olduğundan,
 AB , ΔE kenarına,
ve $A\Gamma$, ΔH kenarına,
 BA ve $A\Gamma$ ikilisi,
 $E\Delta$ ve ΔH ikilisine,
eşittirler,
her biri birine;
ve BAG açısı
 $E\Delta H$ açısına eşittir;
dolayısıyla $B\Gamma$ tabanı
 EH tabanına eşittir.
Dahası,
eşit olduğundan,
 ΔZ , ΔH kenarına,
yine eşittir,
 ΔHZ açısı, ΔZH açısına;
dolayısıyla büyüktür
 ΔZH , EZH açısından;
dolayısıyla çok daha büyüktür
 EZH , EHZ açısından.
Ve EZH bir üçgen olduğundan,
büyük olan
 EZH açısı EZH açısından,
ve daha büyük açı,
—daha büyük açı tarafından karşı-
landığından;
büyüktür dolayısıyla
 EH kenarı da EZ kenarından.
Ve eşittir, EH , $B\Gamma$ kenarına;
büyüktür dolayısıyla $B\Gamma$, EZ kenarın-
dan.

If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
but angle
than angle
have greater,
[namely] that by the equal sides
contained,
also base
than base
they will have greater;
—just what it was necessary to show.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο πλευρὰς
δυσὶ πλευραῖς
ἴσας ἔχῃ
ἑκατέραν ἑκατέρα,
τὴν δὲ γωνίαν
τῆς γωνίας
μείζονα ἔχῃ
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν
τῆς βάσεως
μείζονα ἔξει·
ὅπερ ἔδει δεῖξαι.

Eğer, dolayısıyla, iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına
eşitse
her biri birine,
ama açısı
açısından
büyükse,
[yani] eşit kenarlarca
içerilen(ler),
tabanı da
tabanından
büyük olacak;
—gösterilmesi gereken tam buydu.



1.25

If two triangles
two sides
to two sides
have equal,
either to either,
but base
than base
have greater,
also angle
than angle
they will have greater
—that by the equal STRAIGHTS
contained.

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
δυσὶ πλευραῖς
ἴσας ἔχῃ
ἑκατέραν ἑκατέρα,
τὴν δὲ βάσιν
τῆς βάσεως
μείζονα ἔχῃ,
καὶ τὴν γωνίαν
τῆς γωνίας
μείζονα ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην.

Eğer iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına
eşitse
her biri birine,
ama tabanı
tabanından
büyükse,
açısı da
açısından
büyük olacak
—(yani) eşit doğrularca
içerilenler.

Let there be
two triangles, $AB\Gamma$ and ΔEZ ,
two sides, AB and $A\Gamma$,
to two sides, ΔE and ΔZ ,
having equal,
either to either,
 AB to ΔE
and $A\Gamma$ to ΔZ ;
and the base $B\Gamma$
than the base EZ
—let it be greater.

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , $A\Gamma$
ταῖς δύο πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἑκατέραν ἑκατέρα,
τὴν μὲν AB τῇ ΔE ,
τὴν δὲ $A\Gamma$ τῇ ΔZ ·
βάσις δὲ ἡ $B\Gamma$
βάσεως τῆς EZ
μείζων ἔστω·

Verilmiş olsun
 $AB\Gamma$ ve ΔEZ üçgenleri,
iki AB ve $A\Gamma$ kenarı,
iki ΔE ve ΔZ kenarına,
eşit olan,
her biri birine,
 AB , ΔE kenarına
ve $A\Gamma$, ΔZ kenarına;
ve $B\Gamma$ tabanı
 EZ tabanından
—büyük olsun.

I say that
also the angle BAG
than the angle $E\Delta Z$
is greater.

λέγω, ὅτι
καὶ γωνία ἡ ὑπὸ BAG
γωνίας τῆς ὑπὸ $E\Delta Z$
μείζων ἔστί·

İddia ediyorum ki
 BAG açısı da
 $E\Delta Z$ açısından
büyüktür.

For if not,
[it] is either equal to it, or less;
but it is not equal
— BAG to $E\Delta Z$;
for if it is equal,
also the base $B\Gamma$ to EZ ;

Εἰ γὰρ μή,
ἤτοι ἴση ἔστιν αὐτῇ ἢ ἐλάσσων·
ἴση μὲν οὖν οὐκ ἔστιν
ἢ ὑπὸ BAG τῇ ὑπὸ $E\Delta Z$ ·
ἴση γὰρ ἂν ἦν
καὶ βάσις ἡ $B\Gamma$ βάσει τῇ EZ ·

Çünkü eğer değilse,
ya ona eşittir, ya da ondan küçük;
ama eşit değildir
— BAG , $E\Delta Z$ açısına;
çünkü eğer eşit ise,
 $B\Gamma$ tabanı da EZ tabanına (eşittir);

but it is not.
Therefore it is not equal,
angle $B\hat{A}G$ to $E\hat{\Delta}Z$;
neither is it less,
 $B\hat{A}G$ than $E\hat{\Delta}Z$;
for if it is less,
also base $B\Gamma$ than EZ ;
but it is not;
therefore it is not less,
 $B\hat{A}G$ than angle $E\hat{\Delta}Z$.
And it was shown that
it is not equal;
therefore it is greater,
 $B\hat{A}G$ than $E\hat{\Delta}Z$.

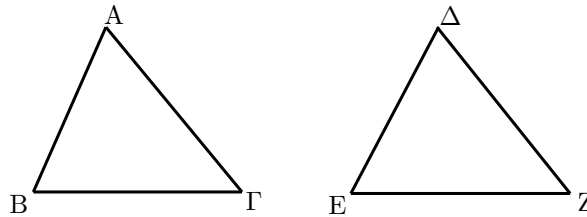
If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
but base
than base
have greater,
also angle
than angle
they will have greater
—that by the equal STRAIGHTS
contained
—just what it was necessary to show.

οὐκ ἔστι δέ.
οὐκ ἄρα ἴση ἐστὶ
γωνία ἢ ὑπὸ $B\hat{A}G$ τῆς ὑπὸ $E\hat{\Delta}Z$.
οὐδὲ μὴν ἐλάσσων ἐστὶν
ἢ ὑπὸ $B\hat{A}G$ τῆς ὑπὸ $E\hat{\Delta}Z$.
ἐλάσσων γὰρ ἂν ἦν
καὶ βᾶσις ἢ $B\Gamma$ βᾶσεως τῆς EZ .
οὐκ ἔστι δέ.
οὐκ ἄρα ἐλάσσων ἐστὶν
ἢ ὑπὸ $B\hat{A}G$ γωνία τῆς ὑπὸ $E\hat{\Delta}Z$.
ἐδείχθη δέ, ὅτι
οὐδὲ ἴση.
μεῖζων ἄρα ἐστὶν
ἢ ὑπὸ $B\hat{A}G$ τῆς ὑπὸ $E\hat{\Delta}Z$.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο πλευρὰς
δυσὶ πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκάτερα,
τὴν δὲ βᾶσιν
τῆς βᾶσεως
μεῖζονα ἔχη,
καὶ τὴν γωνίαν
τῆς γωνίας
μεῖζονα ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην.
ὅπερ ἔδει δεῖξαι.

ama değil.
Dolayısıyla eşit değildir,
 $B\hat{A}G$, $E\hat{\Delta}Z$ açısına;
küçük de değildir,
 $B\hat{A}G$, $E\hat{\Delta}Z$ açısından;
çünkü eğer küçük ise,
 $B\Gamma$ tabanı da EZ tabanından (küçük-
tür);
ama değil;
dolayısıyla küçük değildir,
 $B\hat{A}G$, $E\hat{\Delta}Z$ açısından.
Ama gösterilmişti ki
eşit değildir;
dolayısıyla büyüktür,
 $B\hat{A}G$, $E\hat{\Delta}Z$ açısından.

Eğer, dolayısıyla, iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına
eşitse
her biri birine,
ama tabanı
tabanından
büyükse,
açısı da
açısından
büyük olacak
—(yani) eşit doğrularca
içerilenler;
—gösterilmesi gereken tam buydu.



1.26

If two triangles
two angles
to two angles
have equal,
either to either,
and one side
to one side
equal,
either that near the equal sides
or that subtending
one of the equal sides,
also the remaining sides
to the remaining sides
they will have equal,
also the remaining angle
to the remaining angle.

Let there be
two triangles, $AB\Gamma$ and ΔEZ
the two angles $AB\hat{\Gamma}$ and $B\hat{\Gamma}A$
to the two angles $\Delta E\hat{Z}$ and $E\hat{Z}\Delta$

Ἐὰν δύο τρίγωνα
τὰς δύο γωνίας
δυσὶ γωνίαις
ἴσας ἔχη
ἐκατέραν ἐκάτερα
καὶ μίαν πλευρὰν
μὴ πλευρᾶ
ἴσην
ἢτοι τὴν πρὸς ταῖς ἴσαις γωνίαις
ἢ τὴν ὑποτείνουσαν
ὑπὸ μίαν τῶν ἴσων γωνιῶν,
καὶ τὰς λοιπὰς πλευρὰς
ταῖς λοιπαῖς πλευραῖς
ἴσας ἔξει
καὶ τὴν λοιπὴν γωνίαν
τῆς λοιπῆς γωνίας.

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο γωνίας τὰς ὑπὸ $AB\hat{\Gamma}$, $B\hat{\Gamma}A$
δυσὶ ταῖς ὑπὸ $\Delta E\hat{Z}$, $E\hat{Z}\Delta$

Eğer iki üçgenin
iki açısı
iki açısına
eşitse,
her biri birine,
ve bir kenar
bir kenara
eşitse,
ya eşit açılarının arasında olan
ya da karşılayan
eşit açılardan birini,
kalan kenarları da
kalan kenarlarına
eşit olacak,
kalan açıları da
kalan açılarına.

Verilmiş olsun
iki $AB\Gamma$ ve ΔEZ üçgeni
iki $AB\hat{\Gamma}$ ve $B\hat{\Gamma}A$ açıları
iki $\Delta E\hat{Z}$ ve $E\hat{Z}\Delta$ açılarına

having equal,
either to either,
ABΓ to ΔEZ,
and BΓA to EZΔ;
and let them also have
one side
to one side
equal,
first that near the equal angles,
BΓ to EZ;

I say that
the remaining sides
to the remaining sides
they will have equal,
either to either,
AB to ΔE
and AΓ to ΔZ,
also the remaining angle
to the remaining angle,
BAΓ to EΔZ.

For, if it is unequal,
AB to ΔE,
one of them is greater.
Let be greater
AB,
and let there be cut
to ΔE equal
BH,
and suppose there has been joined
HΓ.

Because then it is equal,
BH to ΔE,
and BΓ to EZ,
the two, BH¹ and BΓ
to the two ΔE and EZ
are equal,
either to either,
and the angle HBG
to the angle ΔEZ
is equal;
therefore the base HΓ
to the base ΔZ
is equal,
and the triangle HBG
to the triangle ΔEZ
is equal,
and the remaining angles
to the remaining angles
will be equal,
those that the equal sides subtend.
Equal therefore is angle BΓH
to ΔZE.
But ΔZE
to BΓA
is supposed equal;
therefore also BΓH
to BΓA
is equal,
the lesser to the greater,
which is impossible.

ἴσας ἔχοντα
ἑκατέραν ἑκατέρα,
τὴν μὲν ὑπὸ ABΓ τῆ ὑπὸ ΔEZ,
τὴν δὲ ὑπὸ BΓA τῆ ὑπὸ EZΔ·
ἔχέτω δὲ
καὶ μίαν πλευρὰν
μῆ πλευρᾶ
ἴσην,
πρότερον τὴν πρὸς ταῖς ἴσαις γωνίαις
τὴν BΓ τῆ EZ·

λέγω, ὅτι
καὶ τὰς λοιπὰς πλευρὰς
ταῖς λοιπαῖς πλευραῖς
ἴσας ἔξει
ἑκατέραν ἑκατέρα,
τὴν μὲν AB τῆ ΔE
τὴν δὲ AΓ τῆ ΔZ,
καὶ τὴν λοιπὴν γωνίαν
τῆ λοιπῆ γωνία,
τὴν ὑπὸ BAΓ τῆ ὑπὸ EΔZ.

Εἰ γὰρ ἄνισός ἐστιν
ἢ AB τῆ ΔE,
μία αὐτῶν μείζων ἐστίν.
ἔστω μείζων
ἢ AB,
καὶ κείσθω
τῆ ΔE ἴση
ἢ BH,
καὶ ἐπεζεύχθω
ἢ HΓ.

Ἐπεὶ οὖν ἴση ἐστίν
ἢ μὲν BH τῆ ΔE,
ἢ δὲ BΓ τῆ EZ,
δύο δὴ αἱ BH, BΓ
δυσὶ ταῖς ΔE, EZ
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρα·
καὶ γωνία ἢ ὑπὸ HBG
γωνία τῆ ὑπὸ ΔEZ
ἴση ἐστίν·
βάσις ἄρα ἢ HΓ
βάσει τῆ ΔZ
ἴση ἐστίν,
καὶ τὸ HBG τρίγωνον
τῷ ΔEZ τριγώνῳ
ἴσον ἐστίν,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσσονται,
ὅψ' ἄς αἱ ἴσαι πλευραὶ ὑποτείνουσιν·
ἴση ἄρα ἢ ὑπὸ BΓH γωνία
τῆ ὑπὸ ΔZE.
ἀλλὰ ἢ ὑπὸ ΔZE
τῆ ὑπὸ BΓA
ὑπόκειται ἴση·
καὶ ἢ ὑπὸ BΓH ἄρα
τῆ ὑπὸ BΓA
ἴση ἐστίν,
ἢ ἐλάσσων τῆ μείζονι·
ὄπερ ἀδύνατον.

eşit olan,
her biri birine,
ABΓ, ΔEZ açısına
ve BΓA, EZΔ açısına;
ayrıca olsun
bir kenarı
bir kenarına
eşit,
önce eşit açılardan yanın da olan,
BΓ, EZ kenarına;

İddia ediyorum ki
kalan kenarlar
kalan kenarlara
eşit olacaklar,
her biri birine,
AB, ΔE kenarına
ve AΓ, ΔZ kenarına,
ayrıca kalan açı
kalan açılara,
BAΓ, EΔZ açısına.

Çünkü, eğer eşit değilse,
AB, ΔE kenarına,
biri daha büyüktür.
Büyük olan
AB olsun,
ve kesilmiş olsun
ΔE kenarına eşit
BH,
ve birleştirilmiş olsun
HΓ.

Çünkü o zaman eşittir,
BH, ΔE kenarına
ve BΓ, EZ kenarına,
BH ve BΓ ikilisi
ΔE ve EZ ikilisine
eşittirler,
her biri birine,
ve HBG açısı
ΔEZ açısına
eşittir;
dolayısıyla HΓ tabanı
ΔZ tabanına
eşittir,
ve HBG üçgeni
ΔEZ üçgenine
eşittir,
ve kalan açılar
kalan açılara
eşit olacaklar,
eşit kenarların karşılıkları.
Eşittir dolayısıyla BΓH açısı
ΔZE açısına.
Ama ΔZE,
BΓA açısına
eşit kabul edilmişti
dolayısıyla BΓH de
BΓA açısına
eşittir,
daha küçük olan daha büyük olana,
ki bu imkansızdır.

Therefore it is not unequal,
AB to ΔE .

Therefore it is equal.

It is also the case that
B Γ to EZ is equal;
then the two AB and B Γ
to the two ΔE and EZ
are equal,
either to either;
also the angle AB Γ
to the angle ΔEZ
is equal;
therefore the base A Γ
to the base ΔZ
is equal,
and the remaining angle BA Γ
to the remaining angle E ΔZ
is equal.

But then again let them be
—[those angles] equal sides
subtending—

equal,

as AB to ΔE ;

I say again that

also the remaining sides
to the remaining sides
will be equal,
A Γ to ΔZ ,
and B Γ to EZ,
and also the remaining angle BA Γ
to the remaining angle E ΔZ
is equal.

For, if it is unequal,

B Γ to EZ,

one of them is greater.

Let be greater,

if possible,

B Γ ,

and let there be cut

to EZ equal

B Θ ,

and suppose there has been joined
A Θ .

Because also it is equal

—B Θ to EZ

and AB to ΔE ,

then the two AB and B Θ

to the two ΔE and EZ

are equal,

either to either;

and they contain equal angles;

therefore the base A Θ

to the base ΔZ

is equal,

and the triangle AB Θ

to the triangle ΔEZ

οὐκ ἄρα ἄνισός ἐστιν

ἢ AB τῆ ΔE .

ἴση ἄρα.

ἔστι δὲ καὶ

ἢ B Γ τῆ EZ ἴση·

δύο δὲ αἱ AB, B Γ

δυοὶ ταῖς ΔE , EZ

ἴσαι εἰσὶν

ἑκατέρω ἑκατέρω·

καὶ γωνία ἡ ὑπὸ AB Γ

γωνία τῆ ὑπὸ ΔEZ

ἐστὶν ἴση·

βάσις ἄρα ἡ A Γ

βάσει τῆ ΔZ

ἴση ἐστίν,

καὶ λοιπὴ γωνία ἡ ὑπὸ BA Γ

τῆ λοιπῆ γωνία τῆ ὑπὸ E ΔZ

ἴση ἐστίν.

Ἄλλὰ δὲ πάλιν ἔστωσαν

αἱ ὑπὸ τὰς ἴσας γωνίας πλευραὶ

ὑποτείνουσαι

ἴσαι,

ὡς ἡ AB τῆ ΔE ·

λέγω πάλιν, ὅτι

καὶ αἱ λοιπαὶ πλευραὶ

ταῖς λοιπαῖς πλευραῖς

ἴσαι ἔσονται,

ἢ μὲν A Γ τῆ ΔZ ,

ἢ δὲ B Γ τῆ EZ

καὶ ἔτι ἡ λοιπὴ γωνία ἡ ὑπὸ BA Γ

τῆ λοιπῆ γωνία τῆ ὑπὸ E ΔZ

ἴση ἐστίν.

Εἰ γὰρ ἄνισός ἐστιν

ἢ B Γ τῆ EZ,

μία αὐτῶν μείζων ἐστίν.

ἔστω μείζων,

εἰ δυνατόν,

ἢ B Γ ,

καὶ κείσθω

τῆ EZ ἴση

ἢ B Θ ,

καὶ ἐπεζεύχθω

ἢ A Θ .

καὶ ἐπεὶ ἴση ἐστίν

ἢ μὲν B Θ τῆ EZ

ἢ δὲ AB τῆ ΔE ,

δύο δὲ αἱ AB, B Θ

δυοὶ ταῖς ΔE , EZ

ἴσαι εἰσὶν

ἑκατέρω ἑκατέρω·

καὶ γωνίας ἴσας περιέχουσιν·

βάσις ἄρα ἡ A Θ

βάσει τῆ ΔZ

ἴση ἐστίν,

καὶ τὸ AB Θ τρίγωνον

τῷ ΔEZ τριγώνω

Dolayısıyla değildir eşit değil,

AB, ΔE kenarına.

Dolayısıyla eşittir.

Ayrıca durum şöyledir;

B Γ , EZ kenarına eşittir;

o zaman AB ve B Γ ikilisi

ΔE ve EZ ikilisine

eşittirler,

her biri birine;

AB Γ açısı da

ΔEZ açısına

eşittir;

dolayısıyla A Γ tabanı

ΔZ tabanına

eşittir,

ve kalan BA Γ açısı

kalan E ΔZ açısına

eşittir.

Ama o zaman, yine olsunlar

— kenarlar eşit [açıları]

karşılıyan—

eşit,

AB, ΔE kenarına gibi;

Yine iddia ediyorum ki

kalan kenarlar da

kalan kenarlara

eşit olacaklar,

A Γ , ΔZ kenarına

ve B Γ , EZ kenarına

ve kalan BA Γ açısı da

kalan E ΔZ açısına

eşittir.

Çünkü, eğer eşit değil ise,

B Γ , EZ kenarına,

biri daha büyüktür.

Daha büyük olsun,

eğer mümkünse,

B Γ ,

ve kesilmiş olsun

EZ kenarına eşit

B Θ ,

ve kabul edilsin birleştirilmiş olduğu

A Θ kenarının.

Ayrıca eşit olduğundan

—B Θ , EZ kenarına

ve AB, ΔE kenarına

AB ve B Θ ikilisi

ΔE ve EZ ikilisine

eşittirler,

her biri birine;

ama içerirler eşit açıları;

dolayısıyla A Θ tabanı

ΔZ tabanına

eşittir,

ve AB Θ üçgeni

ΔEZ üçgenine

¹Fitzpatrick considers this way of denoting the line to be a 'mistake'; apparently he thinks Euclid should (and perhaps did originally) write HB, for parallelism with ΔE . But HB and BH are the same line, and for all we know, Euclid preferred to write BH because it was in alphabetical order. Netz [12, Ch. 2] studies the general

Greek mathematical practice of using the letters in different order for the same mathematical object. He concludes that changes in order are made on purpose, though he does not address examples like the present one.

is equal,
and the remaining angles
to the remaining angles
are equal,
which the equal sides
subtend.
Therefore equal is
angle $B\theta A$
to $EZ\Delta$.
But $EZ\Delta$
to $B\Gamma A$
is equal;
then of triangle $A\theta\Gamma$
the exterior angle $B\theta A$
is equal
to the interior and opposite
 $B\Gamma A$;
which is impossible.
Therefore it is not unequal,
 $B\Gamma$ to EZ ;
therefore it is equal.
And it is also,
 AB ,
to ΔE ,
equal.
Then the two AB and $B\Gamma$
to the two ΔE and EZ
are equal,
either to either;
and equal angles
they contain;
therefore the base $A\Gamma$
to the base ΔZ
is equal,
and triangle $AB\Gamma$
to triangle ΔEZ
is equal,
and the remaining angle BAG
to the remaining angle $E\Delta Z$
is equal.

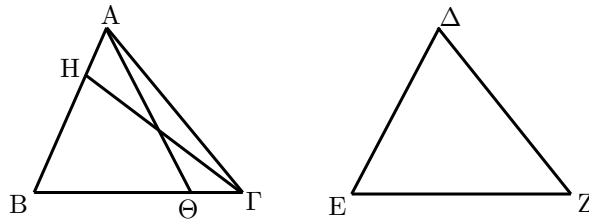
If therefore two triangles
two angles
to two angles
have equal,
either to either,
and one side
to one side
equal,
either that near the equal sides
or that subtending
one of the equal sides,
also the remaining sides
to the remaining sides
they will have equal,
also the remaining angle
to the remaining angle;
—just what it was necessary to show.

ἴσον ἐστίν,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται,
ὅψ' ἂς αἱ ἴσας πλευραὶ
ὑποτείνουσιν·
ἴση ἄρα ἐστὶν
ἡ ὑπὸ $B\theta A$ γωνία
τῇ ὑπὸ $EZ\Delta$.
ἀλλὰ ἡ ὑπὸ $EZ\Delta$
τῇ ὑπὸ $B\Gamma A$
ἐστὶν ἴση·
τριγώνου δὲ τοῦ $A\theta\Gamma$
ἡ ἐκτὸς γωνία ἡ ὑπὸ $B\theta A$
ἴση ἐστὶ
τῇ ἐντὸς καὶ ἀπεναντίον
τῇ ὑπὸ $B\Gamma A$ ·
ὅπερ ἀδύνατον.
οὐκ ἄρα ἀνίσος ἐστὶν
ἡ $B\Gamma$ τῇ EZ ·
ἴση ἄρα.
ἐστὶ δὲ καὶ
ἡ AB
τῇ ΔE
ἴση.
δύο δὲ αἱ AB , $B\Gamma$
δύο ταῖς ΔE , EZ
ἴσαι εἰσὶν
ἐκατέρω ἐκατέρω·
καὶ γωνίας ἴσας
περιέχουσι·
βάσις ἄρα ἡ $A\Gamma$
βάσει τῇ ΔZ
ἴση ἐστίν,
καὶ τὸ $AB\Gamma$ τρίγωνον
τῷ ΔEZ τριγώνῳ
ἴσον
καὶ λοιπὴ γωνία ἡ ὑπὸ BAG
τῇ λοιπῇ γωνίᾳ τῇ ὑπὸ $E\Delta Z$
ἴση.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο γωνίας
δυσὶ γωνίαις
ἴσας ἔχη
ἐκατέραν ἐκατέρω
καὶ μίαν πλευρὰν
μὴ πλευρᾶ
ἴσην
ἢτοι τὴν πρὸς ταῖς ἴσαις γωνίαις,
ἢ τὴν ὑποτείνουσαν
ὑπὸ μίαν τῶν ἴσων γωνιῶν,
καὶ τὰς λοιπὰς πλευρὰς
ταῖς λοιπαῖς πλευραῖς
ἴσας ἔξει
καὶ τὴν λοιπὴν γωνίαν
τῇ λοιπῇ γωνίᾳ·
ὅπερ εἶδει δεῖξαι.

eşittir,
ve kalan açılar
kalan açılara
eşittirler,
eşit kenarların
karşılıkları.
Dolayısıyla eşittir
 $B\theta A$,
 $EZ\Delta$ açısına.
Ama $EZ\Delta$,
 $B\Gamma A$ açısına
eşittir;
o zaman $A\theta\Gamma$ üçgeninin
 $B\theta A$ dış açısı
eşittir
iç ve karşıt
 $B\Gamma A$ açısına;
ki bu imkansızdır.
Dolayısıyla eşit değil değildir,
 $B\Gamma$, EZ kenarına;
dolayısıyla eşittir.
Ve yine
 AB ,
 ΔE kenarına,
eşittir.
O zaman AB ve $B\Gamma$ ikilisi
 ΔE ve EZ ikilisine
eşittirler,
her biri birine;
eşit açılar
içerirler;
dolayısıyla $A\Gamma$ tabanı
 ΔZ tabanına
eşittir,
ve $AB\Gamma$ üçgeni
 ΔEZ üçgenine
eşittir,
ve kalan BAG açısı
kalan $E\Delta Z$ açısına
eşittir.

Eğer, dolayısıyla, iki üçgenin
iki açısı
iki açısına
eşitse,
her biri birine,
ve bir kenar
bir kenara
eşitse,
ya eşit açılar arasında olan
ya da karşılayan
eşit açılardan birini;
kalan kenarları da
kalan kenarlarına
eşit olacak,
kalan açılar da
kalan açılara;
—gösterilmesi gereken tam buydu.



1.27

If on two STRAIGHTS
a STRAIGHT falling
the alternate angles
equal to one another
make,
parallel will be to one another
the STRAIGHTS.

For, on the two STRAIGHTS
AB and ΓΔ
[suppose] the STRAIGHT falling,
[namely] EZ,
the alternate angles
AEZ and EZΔ
equal to one another
make.

I say that
parallel is AB to ΓΔ.

For if not,
extended,
AB and ΓΔ will meet,
either in the B-Δ parts,
or in the A-Γ.
Suppose they have been extended,
and let them meet
in the B-Δ parts
at H.

Of the triangle HEZ
the exterior angle AEZ
is equal
to the interior and opposite
EZH;
which is impossible.
Therefore it is not [the case] that
AB and ΓΔ,
extended,
meet in the B-Δ parts.
Similarly it will be shown that
neither on the A-Γ.
Those that in neither parts
meet
are parallel;
therefore, parallel is AB to ΓΔ.

If therefore on two STRAIGHTS
a STRAIGHT falling
the alternate angles

Ἐὰν εἰς δύο εὐθείας
εὐθεῖα ἐμπίπτουσα
τὰς ἐναλλάξ γωνίας
ἴσας ἀλλήλαις
ποιῆ,
παράλληλοι ἔσονται ἀλλήλαις
αἱ εὐθεῖαι.

Εἰς γὰρ δύο εὐθείας
τὰς AB, ΓΔ
εὐθεῖα ἐμπίπτουσα
ἡ EZ
τὰς ἐναλλάξ γωνίας
τὰς ὑπὸ AEZ, EZΔ
ἴσας ἀλλήλαις
ποιεῖται·

λέγω, ὅτι
παράλληλός ἐστιν ἡ AB τῇ ΓΔ.

Εἰ γὰρ μή,
ἐκβαλλόμεναι
αἱ AB, ΓΔ συμπεσοῦνται
ἤτοι ἐπὶ τὰ B, Δ μέρη
ἢ ἐπὶ τὰ A, Γ.
ἐκβεβλήσθωσαν
καὶ συμπίπτωσαν
ἐπὶ τὰ B, Δ μέρη
κατὰ τὸ H.
τριγώνου δὲ τοῦ HEZ
ἡ ἐκτὸς γωνία ἡ ὑπὸ AEZ
ἴση ἐστὶ
τῇ ἐντὸς καὶ ἀπεναντίον
τῇ ὑπὸ EZH·
ὅπερ ἐστὶν ἀδύνατον·
οὐκ ἄρα
αἱ AB, ΓΔ
ἐκβαλλόμεναι
συμπεσοῦνται ἐπὶ τὰ B, Δ μέρη.
ὁμοίως δὲ δειχθήσεται, ὅτι
οὐδὲ ἐπὶ τὰ A, Γ·
αἱ δὲ ἐπὶ μηδέτερα τὰ μέρη
συμπίπτουσαι
παράλληλοί εἰσιν·
παράλληλος ἄρα ἐστὶν ἡ AB τῇ ΓΔ.

Ἐὰν ἄρα εἰς δύο εὐθείας
εὐθεῖα ἐμπίπτουσα
τὰς ἐναλλάξ γωνίας

Eğer iki doğru üzerine
düşen bir doğru
ters açıları
birbirine eşit
yaparsa
birbirine paralel olacak
doğrular.

Çünkü, iki doğru üzerine,
AB ve ΓΔ,
[kabul edilsin] düşen,
EZ doğrusunun,
ters
AEZ ve EZΔ açılarını
birbirine eşit
oluşturduğunu.

İddia ediyorum ki
paraleldir AB, ΓΔ doğrusuna.

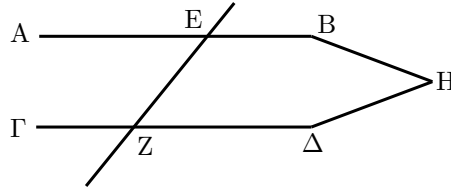
Çünkü eğer değilse,
uzatılmış,
AB ve ΓΔ buluşacaklar,
ya B-Δ parçalarında,
ya da A-Γ parçalarında.
Uzatılmış oldukları kabul edilsin,
ve buluşsunlar
B-Δ parçalarında,
H noktasında.
HEZ üçgeninin
AEZ dış açısı
eşittir
iç ve karşıt
EZH açısına;
ki bu imkansızdır.
Dolayısıyla şöyle değildir (durum)
AB ve ΓΔ,
uzatılmış,
buluşurlar B-Δ parçalarında.
Benzer şekilde gösterilecek ki
A-Γ parçalarında da.
Hiçbir parçada
buluşmayanlar
paraleldir;
dolayısıyla,
paraleldir AB, ΓΔ doğrusuna.

Eğer, dolayısıyla, iki doğru üzerine
düşen bir doğru
ters açıları

equal to one another
make,
parallel will be to one another
the STRAIGHTS;
—just what it was necessary to show.

ἴσας ἀλλήλαις
ποιῆ,
παράλληλοι ἔσσονται
αἱ εὐθεῖαι·
ὅπερ ἔδει δεῖξαι.

birbirine eşit
yaparsa
birbirine paralel olacak
doğrular;
—gösterilmesi gereken tam buydu.



1.28

If on two STRAIGHTS
a STRAIGHT falling¹
the exterior angle
to the interior and opposite
and in the same parts
make equal,
or the interior and in the same parts
to two RIGHTS
equal,
parallel will be to one another
the STRAIGHTS.

For, on the two STRAIGHTS AB and
ΓΔ,
the STRAIGHT falling—EZ—
the exterior angle EHB
to the interior and opposite angle
HΘΔ
equal
—suppose it makes,
or the interior and in the same parts,
BHΘ and HΘΔ,
to two RIGHTS
equal.

I say that
parallel is
AB to ΓΔ.

For, since equal is
EHB to HΘΔ,
while EHB to AHΘ
is equal,
therefore also AHΘ to HΘΔ
is equal;
and they are alternate;
parallel therefore is AB to ΓΔ.

Alternatively, since BHΘ and HΘΔ
to two RIGHTS
are equal,
and also are AHΘ and BHΘ
to two RIGHTS

Ἐὰν εἰς δύο εὐθείας
εὐθεῖα ἐμπίπτουσα
τὴν ἐκτὸς γωνίαν
τῇ ἐντὸς καὶ ἀπεναντίον
καὶ ἐπὶ τὰ αὐτὰ μέρη
ἴσην ποιῆ
ἢ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
δυσὶν ὀρθαῖς
ἴσας,
παράλληλοι ἔσσονται ἀλλήλαις
αἱ εὐθεῖαι.

Εἰς γὰρ δύο εὐθείας τὰς AB, ΓΔ
εὐθεῖα ἐμπίπτουσα ἡ EZ
τὴν ἐκτὸς γωνίαν τὴν ὑπὸ EHB
τῇ ἐντὸς καὶ ἀπεναντίον γωνίᾳ
τῇ ὑπὸ HΘΔ
ἴσην
ποιεῖτω
ἢ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
τὰς ὑπὸ BHΘ, HΘΔ
δυσὶν ὀρθαῖς
ἴσας·

λέγω, ὅτι
παράλληλός ἐστιν
ἡ AB τῇ ΓΔ.

Ἐπεὶ γὰρ ἴση ἐστὶν
ἡ ὑπὸ EHB τῇ ὑπὸ HΘΔ,
ἀλλὰ ἡ ὑπὸ EHB τῇ ὑπὸ AHΘ
ἐστὶν ἴση,
καὶ ἡ ὑπὸ AHΘ ἄρα τῇ ὑπὸ HΘΔ
ἐστὶν ἴση·
καὶ εἰσὶν ἐναλλάξ·
παράλληλος ἄρα ἐστὶν ἡ AB τῇ ΓΔ.

Πάλιν, ἐπεὶ αἱ ὑπὸ BHΘ, HΘΔ
δύο ὀρθαῖς
ἴσαι εἰσὶν,
εἰσὶ δὲ καὶ αἱ ὑπὸ AHΘ, BHΘ
δυσὶν ὀρθαῖς

Eğer iki doğru üzerine
düşen bir doğru,
dış açıyı,
iç ve karşıt
ve aynı tarafta kalan açıya,
eşit yaparsa,
veya iç ve aynı tarafta kalanları,
iki dik açıya
eşit,
birbirine paralel olacak
doğrular.

Çünkü, AB ve ΓΔ doğruları üzerine
düşen EZ doğrusu
EHB dış açısını
iç ve karşıt
HΘΔ açısına
eşit
—yaptığı varsayalım,
veya iç ve aynı tarafta kalan,
BHΘ ve HΘΔ açılarının,
iki dik açıya
eşit olduğu.

İddia ediyorum ki
paraleldir
AB, ΓΔ doğrusuna.

Çünkü, eşit olduğundan
EHB, HΘΔ açısına,
aynı zamanda EHB, AHΘ açısına
eşitken,
dolayısıyla AHΘ de HΘΔ açısına
eşittir;
ve terstirler;
paraleldirler dolayısıyla AB ve ΓΔ.

Ya da BHΘ ve HΘΔ,
iki dik açıya
eşittir,
ve AHΘ ve BHΘ de
iki dik açıya

¹It is perhaps impossible to maintain the Greek word order comprehensibly in English. The normal English order would be, 'If a straight line, falling on two straight lines'. But the proposition is

ultimately about the *two* straight lines; perhaps that is why Euclid mentions them before the one straight line that falls on them.

equal,
therefore $AH\theta$ and $BH\theta$
to $BH\theta$ and $H\theta\Delta$
are equal;
suppose the common has been taken
away
— $BH\theta$;
therefore the remaining $AH\theta$
to the remaining $H\theta\Delta$
is equal;
also they are alternate;
parallel therefore are AB and $\Gamma\Delta$.

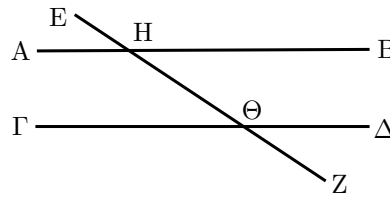
If therefore on two STRAIGHTS
a STRAIGHT falling
the exterior angle
to the interior and opposite
and in the same parts
make equal,
or the interior and in the same parts
to two RIGHTS
equal,
parallel will be to one another
the STRAIGHTS;
—just what it was necessary to show.

ἴσαι,
αἱ ἄρα ὑπὸ $AH\theta$, $BH\theta$
ταῖς ὑπὸ $BH\theta$, $H\theta\Delta$
ἴσαι εἰσίν·
κοινὴ ἀφαιρεθῶ
ἡ ὑπὸ $BH\theta$ ·
λοιπὴ ἄρα ἡ ὑπὸ $AH\theta$
λοιπῇ τῇ ὑπὸ $H\theta\Delta$
ἐστὶν ἴση·
καὶ εἰσὶν ἐναλλάξ·
παράλληλος ἄρα ἐστὶν ἡ AB τῇ $\Gamma\Delta$.

Ἐὰν ἄρα εἰς δύο εὐθείας
εὐθεῖα ἐμπίπτουσα
τὴν ἐκτὸς γωνίαν
τῇ ἐντὸς καὶ ἀπεναντίον
καὶ ἐπὶ τὰ αὐτὰ μέρη
ἴσην ποιῇ
ἢ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
δυσὶν ὀρθαῖς
ἴσας,
παράλληλοι ἔσονται
αἱ εὐθεῖαι·
ὅπερ ἔδει δεῖξαι.

eşittir,
dolayısıyla $AH\theta$ ve $BH\theta$,
 $BH\theta$ ve $H\theta\Delta$ açılara
eşittir;
varsayalım çıkartılmış olduğu ortak
olan
 $BH\theta$ açısının;
dolayısıyla $AH\theta$ kalanı
 $H\theta\Delta$ kalanına
eşittir
ve bunlar terstirler;
paraleldir dolayısıyla AB ve $\Gamma\Delta$.

Eğer dolayısıyla iki doğru üzerine
düşen bir doğru,
dış açıyı,
iç ve karşıt
ve aynı tarafta kalan açıya,
eşit yaparsa,
veya iç ve aynı tarafta kalanları,
iki dik açıya
eşit,
birbirine paralel olacak
doğrular; —gösterilmesi gereken tam
buydu.



1.29

The STRAIGHT falling on parallel
STRAIGHTS
the alternate angles
makes equal to one another,
and the exterior
to the interior and opposite
equal,
and the interior and in the same parts
to two RIGHTS equal.

For, on the parallel STRAIGHTS
 AB and $\Gamma\Delta$
let the STRAIGHT EZ fall.

I say that
the alternate angles
 $AH\theta$ and $H\theta\Delta$
equal
it makes,
and the exterior angle EHB
to the interior and opposite $H\theta\Delta$
equal,
and the interior and in the same parts
 $BH\theta$ and $H\theta\Delta$
to two RIGHTS equal.

Ἡ εἰς τὰς παραλλήλους εὐθείας εὐθεῖα
ἐμπίπτουσα
τάς τε ἐναλλάξ γωνίας
ἴσας ἀλλήλαις ποιεῖ
καὶ τὴν ἐκτὸς
τῇ ἐντὸς καὶ ἀπεναντίον
ἴσην
καὶ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
δυσὶν ὀρθαῖς ἴσας.

Εἰς γὰρ παραλλήλους εὐθείας
τὰς AB , $\Gamma\Delta$
εὐθεῖα ἐμπίπτέτω ἡ EZ .

λέγω, ὅτι
τὰς ἐναλλάξ γωνίας
τὰς ὑπὸ $AH\theta$, $H\theta\Delta$
ἴσας
ποιεῖ
καὶ τὴν ἐκτὸς γωνίαν τὴν ὑπὸ EHB
τῇ ἐντὸς καὶ ἀπεναντίον τῇ ὑπὸ $H\theta\Delta$
ἴσην
καὶ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
τὰς ὑπὸ $BH\theta$, $H\theta\Delta$
δυσὶν ὀρθαῖς ἴσας.

Paralel doğrular üzerine düşen bir
doğru
ters açıları
birbirine eşit yapar,
ve dış açıyı
iç ve karşıt açıya
eşit,
ve iç ve aynı tarafta kalanları
iki dik açıya eşit.

Çünkü, paralel
 AB ve $\Gamma\Delta$ doğruları üzerine
 EZ doğrusu düşsün.

İddia ediyorum ki
ters
 $AH\theta$ ve $H\theta\Delta$ açılarını
eşit
yapar,
ve EHB dış açısını
iç ve karşıt $H\theta\Delta$ açısına
eşit,
ve iç ve aynı taraftaki
 $BH\theta$ ile $H\theta\Delta$ açılarını
iki dik açıya eşit.

For, if it is unequal,
 $AH\theta$ to $H\theta\Delta$,
 one of them is greater.
 Let the greater be $AH\theta$;
 let be added in common
 $BH\theta$;
 therefore $AH\theta$ and $BH\theta$
 than $BH\theta$ and $H\theta\Delta$
 are greater.
 However, $AH\theta$ and $BH\theta$
 to two RIGHTS
 equal are.
 Therefore [also] $BH\theta$ and $H\theta\Delta$
 than two RIGHTS
 less are.
 And [STRAIGHTS] from [angles] that
 are less
 than two RIGHTS,
 extended to the infinite,
 fall together.
 Therefore AB and $\Gamma\Delta$,
 extended to the infinite,
 will fall together.
 But they do not fall together,
 by their being assumed parallel.
 Therefore is not unequal
 $AH\theta$ to $H\theta\Delta$.
 Therefore it is equal.
 However, $AH\theta$ to EHB
 is equal;
 therefore also EHB to $H\theta\Delta$
 is equal;
 let $BH\theta$ be added in common;
 therefore EHB and $BH\theta$
 to $BH\theta$ and $H\theta\Delta$
 is equal.
 But EHB and $BH\theta$
 to two RIGHTS
 are equal.
 Therefore also $BH\theta$ and $H\theta\Delta$
 to two RIGHTS
 are equal.

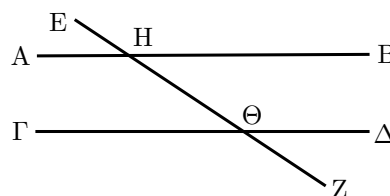
Therefore the on-parallel-STRAIGHTS
 STRAIGHT
 falling
 the alternate angles
 makes equal to one another,
 and the exterior
 to the interior and opposite
 equal,
 and the interior and in the same parts
 to two RIGHTS equal;
 —just what it was necessary to show.

Εἰ γὰρ ἄνισός ἐστιν
 ἢ ὑπὸ $AH\theta$ τῆ ὑπὸ $H\theta\Delta$,
 μία αὐτῶν μείζων ἐστίν.
 ἔστω μείζων ἢ ὑπὸ $AH\theta$.
 κοινῇ προσκείσθω
 ἢ ὑπὸ $BH\theta$.
 αἱ ἄρα ὑπὸ $AH\theta$, $BH\theta$
 τῶν ὑπὸ $BH\theta$, $H\theta\Delta$
 μείζονές εἰσιν.
 ἀλλὰ αἱ ὑπὸ $AH\theta$, $BH\theta$
 δυσὶν ὀρθαῖς
 ἴσαι εἰσίν.
 [καὶ] αἱ ἄρα ὑπὸ $BH\theta$, $H\theta\Delta$
 ὀρθῶν
 ἐλάσσονές εἰσιν.
 αἱ δὲ ἀπ' ἐλασσόνων
 ἢ ὀρθῶν
 ἐκβαλλόμεναι
 εἰς ἄπειρον
 συμπίπτουσιν.
 αἱ ἄρα AB , $\Gamma\Delta$
 ἐκβαλλόμεναι εἰς ἄπειρον
 συμπεσοῦνται.
 οὐ συμπίπτουσι δὲ
 διὰ τὸ παραλλήλους αὐτὰς ὑποκεῖσθαι.
 οὐκ ἄρα ἄνισός ἐστιν
 ἢ ὑπὸ $AH\theta$ τῆ ὑπὸ $H\theta\Delta$.
 ἴση ἄρα.
 ἀλλὰ ἢ ὑπὸ $AH\theta$ τῆ ὑπὸ EHB
 ἐστὶν ἴση.
 καὶ ἢ ὑπὸ EHB ἄρα τῆ ὑπὸ $H\theta\Delta$
 ἐστὶν ἴση.
 κοινῇ προσκείσθω ἢ ὑπὸ $BH\theta$.
 αἱ ἄρα ὑπὸ EHB , $BH\theta$
 ταῖς ὑπὸ $BH\theta$, $H\theta\Delta$
 ἴσαι εἰσίν.
 ἀλλὰ αἱ ὑπὸ EHB , $BH\theta$
 δύο ὀρθαῖς
 ἴσαι εἰσίν.
 καὶ αἱ ὑπὸ $BH\theta$, $H\theta\Delta$ ἄρα
 δύο ὀρθαῖς
 ἴσαι εἰσίν.

ἼΗ ἄρα εἰς τὰς παραλλήλους εὐθείας εὐ-
 θεῖα
 ἐπίπτουσα
 τὰς τε ἐναλλάξ γωνίας
 ἴσας ἀλλήλαις ποιεῖ
 καὶ τὴν ἐκτὸς
 τῆ ἐντὸς καὶ ἀπεναντίον
 ἴσην
 καὶ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
 δυσὶν ὀρθαῖς ἴσας.
 ὅπερ ἔδει δεῖξαι.

Çünkü, eğer eşit değilse
 $AH\theta$, $H\theta\Delta$ açısına,
 biri büyüktür.
 Büyük olan $AH\theta$ olsun;
 eklenmiş olsun her ikisine de
 $BH\theta$;
 dolayısıyla $AH\theta$ ve $BH\theta$,
 $BH\theta$ ve $H\theta\Delta$ açılarından
 büyüktürler.
 Fakat, $AH\theta$ ve $BH\theta$
 iki dik açıya
 eşittirler.
 Dolayısıyla $BH\theta$ ve $H\theta\Delta$ [da]
 iki dik açıdan
 küçüktürler.
 Ve küçük olanlardan,
 iki dik açıdan,
 sonsuza uzatılanlar [doğrular],
 birbirinin üzerine düşerler.
 Dolayısıyla AB ve $\Gamma\Delta$,
 uzatılınca sonsuza,
 birbirinin üzerine düşecekler.
 Ama onlar birbirinin üzerine düşme-
 zler,
 paralel oldukları kabul edildiğinden.
 Dolayısıyla eşit değil değildir
 $AH\theta$, $H\theta\Delta$ açısına.
 Dolayısıyla eşittir.
 Ancak, $AH\theta$, EHB açısına
 eşittir;
 dolayısıyla EHB da $H\theta\Delta$ açısına
 eşittir;
 eklenmiş olsun her ikisine de $BH\theta$;
 dolayısıyla EHB ve $BH\theta$,
 $BH\theta$ ve $H\theta\Delta$ açılara
 eşittir.
 Ama EHB ve $BH\theta$
 iki dik açıya
 eşittirler.
 Dolayısıyla $BH\theta$ ve $H\theta\Delta$ da
 iki dik açıya
 eşittirler.

Dolayısıyla paralel doğrular üzerine,
 doğru
 düşerken
 ters açıları
 eşit yapar birbirine,
 ve dış açıyı
 iç ve karşıta
 eşit,
 ve iç ve aynı taraftakileri s
 iki dik açıya eşit;
 —gösterilmesi gereken tam buydu.



1.30

[STRAIGHTS] to the same STRAIGHT parallel also to one another are parallel.

Let be either of AB and ΓΔ to ΓΔ parallel.

I say that also AB to ΓΔ is parallel.

For let fall on them a STRAIGHT, HK.

Then, since on the parallel STRAIGHTS AB and EZ a STRAIGHT has fallen, [namely] HK, equal therefore is AHK to HΘZ. Moreover, since on the parallel STRAIGHTS EZ and ΓΔ a STRAIGHT has fallen, [namely] HK, equal is HΘZ to HKΔ. And it was shown also that AHK to HΘZ is equal. Also AHK therefore to HKΔ is equal; and they are alternate. Parallel therefore is AB to ΓΔ.

Therefore [STRAIGHTS] to the same STRAIGHT parallel also to one another are parallel; —just what it was necessary to show.

Αἱ τῆ αὐτῆ εὐθεία παράλληλοι καὶ ἀλλήλαις εἰσι παράλληλοι.

Ἐστω ἑκατέρα τῶν AB, ΓΔ τῆ EZ παράλληλος·

λέγω, ὅτι καὶ ἡ AB τῆ ΓΔ ἐστὶ παράλληλος.

Ἐμπίπτετω γὰρ εἰς αὐτὰς εὐθεῖα ἡ HK.

Καὶ ἐπεὶ εἰς παραλλήλους εὐθείας τὰς AB, EZ εὐθεῖα ἐμπίπτωκεν ἡ HK, ἴση ἄρα ἡ ὑπὸ AHK τῆ ὑπὸ HΘZ. πάλιν, ἐπεὶ εἰς παραλλήλους εὐθείας τὰς EZ, ΓΔ εὐθεῖα ἐμπίπτωκεν ἡ HK, ἴση ἐστὶν ἡ ὑπὸ HΘZ τῆ ὑπὸ HKΔ. ἐδείχθη δὲ καὶ ἡ ὑπὸ AHK τῆ ὑπὸ HΘZ ἴση. καὶ ἡ ὑπὸ AHK ἄρα τῆ ὑπὸ HKΔ ἐστὶν ἴση· καὶ εἰσὶν ἐναλλάξ. παράλληλος ἄρα ἐστὶν ἡ AB τῆ ΓΔ.

[Αἱ ἄρα τῆ αὐτῆ εὐθεία παράλληλοι καὶ ἀλλήλαις εἰσι παράλληλοι·] ὅπερ εἶδει δεῖξαι.

Aynı doğruya paralel doğrular birbirlerine de paraleldir.

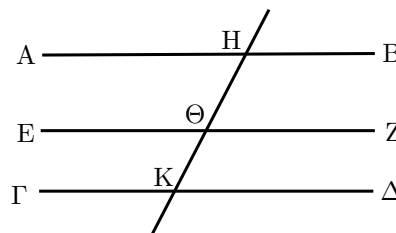
Olsun AB ve ΓΔ doğrularının her biri, ΓΔ doğrusuna paralel.

İddia ediyorum ki AB da ΓΔ doğrusuna paraleldir.

Çünkü üzerlerine bir HK doğrusu düşmüş olsun.

O zaman, paralel AB ve EZ doğrularının üzerine bir doğru düşmüş olduğundan, [yani] HK, eşittir dolayısıyla AHK, HΘZ açısına. Dahası, paralel EZ ve ΓΔ doğrularının üzerine bir doğru düşmüş olduğundan, [yani] HK, eşittir HΘZ, HKΔ açısına. Ve gösterilmişti ki AHK, HΘZ açısına eşittir. Ve AHK dolayısıyla HKΔ açısına eşittir; ve bunlar terstirler. Paraleldir dolayısıyla AB, ΓΔ doğrusuna.

Dolayısıyla aynı doğruya paraleller birbirlerine de paraleldir; —gösterilmesi gereken tam buydu.



1.31

Through the given point to the given STRAIGHT parallel a straight line to draw.

Let be the given point A, and the given STRAIGHT BΓ.

It is necessary then

Διὰ τοῦ δοθέντος σημείου τῆ δοθείση εὐθεία παράλληλον εὐθεῖαν γραμμὴν ἀγαγεῖν.

Ἐστω τὸ μὲν δοθὲν σημεῖον τὸ A, ἡ δὲ δοθεῖσα εὐθεῖα ἡ BΓ·

δεῖ δὴ

Verilen bir noktadan verilen bir doğruya paralel bir doğru çizmek.

Olsun verilen nokta A, ve verilen doğru BΓ.

Şimdi gereklidir

through the point A
to the STRAIGHT BΓ parallel
a straight line to draw.

Suppose there has been chosen
on BΓ
a random point Δ,
and there has been joined AΔ.
and there has been constructed,
on the STRAIGHT ΔA,
and at the point A of it,
to the angle AΔΓ equal,
ΔAE;
and suppose there has been extended,
in STRAIGHTS with EA,
the STRAIGHT AZ.

And because
on the two STRAIGHTS BΓ and EZ
the straight line falling, AΔ,
the alternate angles
EAD and AΔΓ
equal to one another has made,
parallel therefore is EAZ to BΓ.

Therefore, through the given point A,
to the given STRAIGHT BΓ parallel,
a straight line has been drawn, EAZ;
—just what it was necessary to do.

διὰ τοῦ A σημείου
τῆς BΓ εὐθείας παράλληλον
εὐθεῖαν γραμμὴν ἀγαγεῖν.

Εἰλήφθω
ἐπὶ τῆς BΓ
τυχόν σημείον τὸ Δ,
καὶ ἐπεζεύχθω ἡ AΔ·
καὶ συνεστάτω
πρὸς τῆς ΔA εὐθείας
καὶ τῷ πρὸς αὐτῆς σημείῳ τῷ A
τῆς ὑπὸ AΔΓ γωνίας ἴση
ἢ ὑπὸ ΔAE·
καὶ ἐκβεβλήσθω
ἐπ' εὐθείας τῆς EA
εὐθεῖα ἡ AZ.

Καὶ ἐπεὶ
εἰς δύο εὐθείας τὰς BΓ, EZ
εὐθεῖα ἐμπίπτουσα ἡ AΔ
τὰς ἐναλλάξ γωνίας
τὰς ὑπὸ EAD, AΔΓ
ἴσας ἀλλήλαις πεποίηκεν,
παράλληλος ἄρα ἐστὶν ἡ EAZ τῆς BΓ.

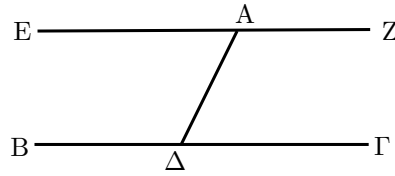
Διὰ τοῦ δοθέντος ἄρα σημείου τοῦ A
τῆς δοθείσης εὐθείας τῆς BΓ παράλληλος
εὐθεῖα γραμμὴ ἤχεται ἡ EAZ·
ὅπερ ἔδει ποιῆσαι.

A noktasından
BΓ doğrusuna paralel
bir doğru çizmek.

Varsayalım seçilmiş olduğu
BΓ üzerinde
rastgele bir Δ noktasının,
ve AΔ doğrusunun birleştirilmiş
olduğu,
ve inşa edilmiş olduğu,
ΔA doğrusunda,
ve onun A noktasında,
AΔΓ açısına eşit,
ΔAE açısının;
ve kabul edilsin uzatılmış olsun,
EA ile aynı doğruda,
AZ doğrusu.

Ve çünkü
BΓ ve EZ doğruları üzerine
düşerken AΔ doğrusu,
ters
EAD ve AΔΓ açılarını
eşit yapmıştır birbirine,
paraleldir dolayısıyla EAZ, BΓ
doğrusuna.

Dolayısıyla, verilen A noktasından,
verilen BΓ doğrusuna paralel,
bir doğru EAZ, çizilmiş oldu;
—yapılması gereken tam buydu.



1.32

Of any triangle
one of the sides being extended,
the exterior angle
to the two opposite interior angles
is equal,
and the triangle's three interior angles
to two RIGHTS equal are.

Let there be
the triangle ABΓ,
and suppose there has been extended
its one side, BΓ, to Δ;

I say that
the exterior angle AΓ is equal

Παντὸς τριγώνου
μῆς τῶν πλευρῶν προσεκβληθείσης
ἡ ἔκτος γωνία
δυσὶ ταῖς ἐντὸς καὶ ἀπεναντίον
ἴση ἐστίν,
καὶ αἱ ἐντὸς τοῦ τριγώνου τρεῖς γωνίαι
δυσὶν ὀρθαῖς ἴσαι εἰσίν.

Ἐστω
τρίγωνον τὸ ABΓ,
καὶ προσεκβεβλήσθω
αὐτοῦ μία πλευρὰ ἡ BΓ ἐπὶ τὸ Δ·

λέγω, ὅτι
ἡ ἔκτος γωνία ἡ ὑπὸ AΓΔ ἴση ἐστὶ

Herhangi bir üçgenin
kenarlarından biri uzatıldığında,
dış açı
iki karşıt iç açıya
eşittir,
ve üçgenin üç iç açısı
iki dik açıya eşittir.

Verilmiş olsun
ABΓ üçgeni,
ve varsayalım uzatılmış olduğu
bir BΓ kenarının Δ noktasına.

İddia ediyorum ki
AΓΔ dış açısı eşittir

to the two interior and opposite angles ΓAB and $AB\Gamma$, and the triangle's three interior angles $AB\Gamma$, $B\Gamma A$, and ΓAB to two RIGHTS equal are.

For, suppose there has been drawn through the point Γ to the STRAIGHT AB parallel ΓE .

And since parallel is AB to ΓE , and on these has fallen $A\Gamma$, the alternate angles $BA\Gamma$ and $A\Gamma E$ equal to one another are. Moreover, since parallel is AB to ΓE , and on these has fallen the STRAIGHT $B\Delta$, the exterior angle $E\Gamma\Delta$ is equal to the interior and opposite $AB\Gamma$. And it was shown that also $A\Gamma E$ to $BA\Gamma$ [is] equal. Therefore the whole angle $A\Gamma\Delta$ is equal to the two interior and opposite angles $BA\Gamma$ and $AB\Gamma$.

Let be added in common $A\Gamma B$; Therefore $A\Gamma\Delta$ and $A\Gamma B$ to the three $AB\Gamma$, $B\Gamma A$, and ΓAB equal are. However, $A\Gamma\Delta$ and $A\Gamma B$ to two RIGHTS equal are; also $A\Gamma B$, $\Gamma B A$, and ΓAB therefore to two RIGHTS equal are.

Therefore, of any triangle one of the sides being extended, the exterior angle to the two opposite interior angles is equal, and the triangle's three interior angles to two RIGHTS equal are; —just what it was necessary to show.

δυσὶ ταῖς ἐντὸς καὶ ἀπεναντίον ταῖς ὑπὸ ΓAB , $AB\Gamma$, καὶ αἱ ἐντὸς τοῦ τριγώνου τρεῖς γωνίαι αἱ ὑπὸ $AB\Gamma$, $B\Gamma A$, ΓAB δυσὶν ὀρθαῖς ἴσαι εἰσίν.

Ἦχθω γὰρ διὰ τοῦ Γ σημείου τῇ AB εὐθείᾳ παράλληλος ἡ ΓE .

Καὶ ἐπεὶ παράλληλός ἐστὶν ἡ AB τῇ ΓE , καὶ εἰς αὐτὰς ἐμπέπτωκεν ἡ $A\Gamma$, αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ $BA\Gamma$, $A\Gamma E$ ἴσαι ἀλλήλαις εἰσίν. πάλιν, ἐπεὶ παράλληλός ἐστὶν ἡ AB τῇ ΓE , καὶ εἰς αὐτὰς ἐμπέπτωκεν εὐθεῖα ἡ $B\Delta$, ἡ ἐκτὸς γωνία ἡ ὑπὸ $E\Gamma\Delta$ ἴση ἐστὶ τῇ ἐντὸς καὶ ἀπεναντίον τῇ ὑπὸ $AB\Gamma$. ἐδείχθη δὲ καὶ ἡ ὑπὸ $A\Gamma E$ τῇ ὑπὸ $BA\Gamma$ ἴση· ὅλη ἄρα ἡ ὑπὸ $A\Gamma\Delta$ γωνία ἴση ἐστὶ δυσὶ ταῖς ἐντὸς καὶ ἀπεναντίον ταῖς ὑπὸ $BA\Gamma$, $AB\Gamma$.

Κοινὴ προσκείσθω ἡ ὑπὸ $A\Gamma B$ · αἱ ἄρα ὑπὸ $A\Gamma\Delta$, $A\Gamma B$ τρισὶ ταῖς ὑπὸ $AB\Gamma$, $B\Gamma A$, ΓAB ἴσαι εἰσίν. ἀλλ' αἱ ὑπὸ $A\Gamma\Delta$, $A\Gamma B$ δυσὶν ὀρθαῖς ἴσαι εἰσίν· καὶ αἱ ὑπὸ $A\Gamma B$, $\Gamma B A$, ΓAB ἄρα δυσὶν ὀρθαῖς ἴσαι εἰσίν.

Παντὸς ἄρα τριγώνου μιᾶς τῶν πλευρῶν προσεκβληθείσης ἡ ἐκτὸς γωνία δυσὶ ταῖς ἐντὸς καὶ ἀπεναντίον ἴση ἐστίν, καὶ αἱ ἐντὸς τοῦ τριγώνου τρεῖς γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσίν· ὅπερ ἔδει δεῖξαι.

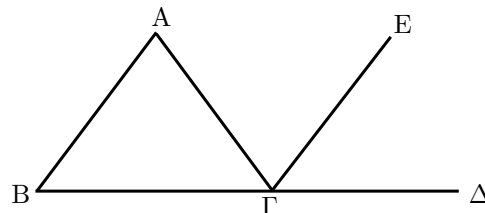
iki iç ve karşıt ΓAB ve $AB\Gamma$ açısına, ve üçgenin üç iç açısı $AB\Gamma$, $B\Gamma A$ ve ΓAB , iki dik açıya eşittir.

Çünkü, varsayalım çizilmiş olduğu Γ noktasından AB doğrusuna paralel ΓE doğrusunun.

Ve paralel olduğundan AB , ΓE doğrusuna, ve bunların üzerine düştüğünden $A\Gamma$, ters $BA\Gamma$ ve $A\Gamma E$ açıları eşittirler birbirlerine. Dahası, paralel olduğundan AB , ΓE doğrusuna, and bunların üzerine düştüğünden $B\Delta$ doğrusu, $E\Gamma\Delta$ dış açısı eşittir iç ve karşıt $AB\Gamma$ açısına. Ve gösterilmişti ki $A\Gamma E$ da $BA\Gamma$ açısına eşittir. Dolayısıyla açının tamamı $A\Gamma\Delta$ eşittir iç ve karşıt $BA\Gamma$ ve $AB\Gamma$ açılarına.

Eklenmiş olsun $A\Gamma B$ ortak olarak; Dolayısıyla $A\Gamma\Delta$ ve $A\Gamma B$ açıları $AB\Gamma$, $B\Gamma A$ ve ΓAB üçlüsüne eşittir. Fakat, $A\Gamma\Delta$ ve $A\Gamma B$ açıları iki dik açıya eşittir; $A\Gamma B$, $\Gamma B A$ ve ΓAB da dolayısıyla iki dik açıya eşittir.

Dolayısıyla, herhangi bir üçgenin kenarlarından biri uzatıldığında, dış açı iki karşıt iç açıya eşittir, ve üçgenin üç iç açısı iki dik açıya eşittir; —gösterilmesi gereken tam buydu.



1.33

STRAIGHTS joining equals and parallels to the same parts also themselves equal and parallel are.

Αἱ τὰς ἴσας τε καὶ παραλλήλους ἐπὶ τὰ αὐτὰ μέρη ἐπιζευγνύουσαι εὐθεῖαι καὶ αὐταὶ ἴσαι τε καὶ παράλληλοί εἰσιν.

Eşit ve paralellerin aynı taraflarını birleştiren doğruların kendileri de eşit ve paraleldirler.

Let be
equals and parallels
AB and ΓΔ,
and let join these
in the same parts
STRAIGHTS ΑΓ and ΒΔ.

I say that
also ΑΓ and ΒΔ
equal and parallel are.

Suppose there has been joined ΒΓ.
And since parallel is AB to ΓΔ,
and on these has fallen ΒΓ,
the alternate angles ΑΒΓ and ΒΓΔ
equal to one another are.
And since equal is AB to ΓΔ,
and common [is] ΒΓ,
then the two ΑΒ and ΒΓ
to the two ΒΓ and ΓΔ
equal are;
also angle ΑΒΓ
to angle ΒΓΔ
[is] equal;
therefore the base ΑΓ
to the base ΒΔ
is equal,
and the triangle ΑΒΓ
to the triangle ΒΓΔ
is equal,
and the remaining angles
to the remaining angles
equal will be,
either to either,
which the equal sides subtend;
equal therefore
the ΑΓΒ angle to ΓΒΔ.
And since on the two STRAIGHTS
ΑΓ and ΒΔ
the STRAIGHT falling—ΒΓ—
alternate angles equal to one another
has made,
parallel therefore is ΑΓ to ΒΔ.
And it was shown to it also equal.

Therefore STRAIGHTS joining equals
and parallels to the same parts
also themselves equal and parallel are.
—just what it was necessary to
show.

Ἐστῶσαν
ἴσαι τε καὶ παράλληλοι
αἱ ΑΒ, ΓΔ,
καὶ ἐπιζευγνύτωσαν αὐτὰς
ἐπὶ τὰ αὐτὰ μέρη
εὐθεῖαι αἱ ΑΓ, ΒΔ.

λέγω, ὅτι
καὶ αἱ ΑΓ, ΒΔ
ἴσαι τε καὶ παράλληλοι εἰσιν.

Ἐπεζεύχθω ἡ ΒΓ.
καὶ ἐπεὶ παράλληλός ἐστιν ἡ ΑΒ τῇ ΓΔ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν ἡ ΒΓ,
αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ ΑΒΓ, ΒΓΔ
ἴσαι ἀλλήλαις εἰσίν.
καὶ ἐπεὶ ἴση ἐστιν ἡ ΑΒ τῇ ΓΔ
κοινὴ δὲ ἡ ΒΓ,
δύο δὴ αἱ ΑΒ, ΒΓ
δύο ταῖς ΒΓ, ΓΔ
ἴσαι εἰσίν·
καὶ γωνία ἡ ὑπὸ ΑΒΓ
γωνία τῇ ὑπὸ ΒΓΔ
ἴση·
βάσις ἄρα ἡ ΑΓ
βάσει τῇ ΒΔ
ἐστὶν ἴση,
καὶ τὸ ΑΒΓ τρίγωνον
τῷ ΒΓΔ τριγώνῳ
ἴσον ἐστίν,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσσονται
ἐκατέρα ἐκατέρᾳ,
ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν·
ἴση ἄρα
ἡ ὑπὸ ΑΓΒ γωνία τῇ ὑπὸ ΓΒΔ.
καὶ ἐπεὶ εἰς δύο εὐθείας
τὰς ΑΓ, ΒΔ
εὐθεῖα ἐμπίπτουσα ἡ ΒΓ
τὰς ἐναλλάξ γωνίας ἴσας ἀλλήλαις
πεποίηκεν,
παράλληλος ἄρα ἐστὶν ἡ ΑΓ τῇ ΒΔ.
ἐδείχθη δὲ αὐτῇ καὶ ἴση.

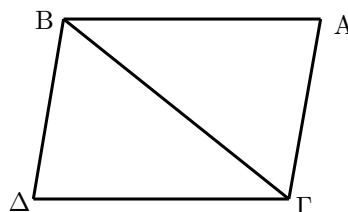
Αἱ ἄρα τὰς ἴσας τε καὶ παραλλήλους ἐ-
πὶ τὰ αὐτὰ μέρη ἐπιζευγνύουσαι
εὐθεῖαι
καὶ αὐταὶ ἴσαι τε καὶ παράλληλοι εἰσιν·
ὅπερ ἔδει δεῖξαι.

Olsun
eşit ve paraleller
AB ve ΓΔ,
ve bunların birleştirsın
aynı taraflarını
ΑΓ ve ΒΔ doğruları.

İddia ediyorum ki
ΑΓ ve ΒΔ da
eşit ve paraleldirler.

Varsayalım birleştirilmiş olduğu ΒΓ
doğrusunun.
Ve paralel olduğundan ΑΒ, ΓΔ
doğrusuna,
ve bunların üzerine düştüğünden ΒΓ,
ters ΑΒΓ ve ΒΓΔ açıları
birbirlerine eşittirler.
Ve eşit olduğundan ΑΒ, ΓΔ
doğrusuna,
ve ΒΓ ortak,
ΑΒ ve ΒΓ ikilisi
ΒΓ ve ΓΔ ikilisine
eşittir;
ΑΒΓ açısı da
ΒΓΔ açısına
eşittir;
dolayısıyla ΑΓ tabanı
ΒΔ tabanına
eşittir,
ve ΑΒΓ üçgeni
ΒΓΔ üçgenine
eşittir,
ve kalan açılar
kalan açılara
eşit olacaklar,
her biri birine,
eşit kenarları görenler;
eşittir dolayısıyla
ΑΓΒ, ΓΒΔ açısına.
Ve üzerine iki
ΑΓ ve ΒΔ doğrularının,
düşen doğru—ΒΓ—
birbirine eşit ters açılar
yapmıştır,
paraleldir dolayısıyla ΑΓ, ΒΔ
doğrusuna.
Ve eşit olduğu da gösterilmişti.

Dolayısıyla eşit ve paralellerin aynı
taraflarını birleştiren doğru-
ların
kendileri de eşit
ve paraleldirler; —gösterilmesi
gerekten tam buydu.



1.34

Of parallelogram areas,
opposite sides and angles
are equal to one another,
and the diameter cuts them in two.

Let there be
a parallelogram area
ΑΓΔΒ;
a diameter of it, ΒΓ.

I say that
of the ΑΓΔΒ parallelogram
the opposite sides and angles
equal to one another are,
and the ΒΓ diameter it cuts in two.

For, since parallel is
ΑΒ to ΓΔ,
and on these has fallen
a STRAIGHT, ΒΓ,
the alternate angles ΑΒΓ and ΒΓΔ
equal to one another are.
Moreover, since parallel is
ΑΓ to ΒΔ,
and on these has fallen
ΒΓ,
the alternate angles ΑΓΒ and ΓΒΔ
equal to one another are.
Then two triangles there are,
ΑΒΓ and ΒΓΔ,
the two angles ΑΒΓ and ΒΓΑ
to the two ΒΓΔ and ΓΒΔ
equal having,
either to either,
and one side to one side equal,
that near the equal angles,
their common ΒΓ;
also then the remaining sides
to the remaining sides
equal they will have,
either to either,
and the remaining angle
to the remaining angle;
equal, therefore,
the ΑΒ side to ΓΔ,
and ΑΓ to ΒΔ,
and yet equal is the ΒΑΓ angle
to ΓΔΒ.
And since equal is the ΑΒΓ angle
to ΒΓΔ,
and ΓΒΔ to ΑΓΒ,
therefore the whole ΑΒΔ
to the whole ΑΓΔ
is equal.
And was shown also
ΒΑΓ to ΓΔΒ equal.

Therefore, of parallelogram areas,
opposite sides and angles
equal to one another are.

Τῶν παραλληλογράμμων χωρίων
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ ἡ διάμετρος αὐτὰ δίχα τέμνει.

Ἐστω
παραλληλόγραμμον χωρίον
τὸ ΑΓΔΒ,
διάμετρος δὲ αὐτοῦ ἡ ΒΓ·

λέγω, ὅτι
τοῦ ΑΓΔΒ παραλληλογράμμου
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ ἡ ΒΓ διάμετρος αὐτὸ δίχα τέμνει.

Ἐπεὶ γὰρ παράλληλός ἐστιν
ἡ ΑΒ τῇ ΓΔ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν
εὐθεῖα ἡ ΒΓ,
αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ ΑΒΓ, ΒΓΔ
ἴσαι ἀλλήλαις εἰσίν.
πάλιν ἐπεὶ παράλληλός ἐστιν
ἡ ΑΓ τῇ ΒΔ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν
ἡ ΒΓ,
αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ ΑΓΒ, ΓΒΔ
ἴσαι ἀλλήλαις εἰσίν.
δύο δὲ τρίγωνά ἐστι
τὰ ΑΒΓ, ΒΓΔ
τὰς δύο γωνίας τὰς ὑπὸ ΑΒΓ, ΒΓΑ
δυσὶ ταῖς ὑπὸ ΒΓΔ, ΓΒΔ
ἴσας ἔχοντα
ἐκατέραν ἐκατέρα
καὶ μίαν πλευρὰν μιᾷ πλευρᾷ ἴσην
τὴν πρὸς ταῖς ἴσαις γωνίαις
κοινὴν αὐτῶν τὴν ΒΓ·
καὶ τὰς λοιπὰς ἄρα πλευρὰς
ταῖς λοιπαῖς
ἴσας ἔξει
ἐκατέραν ἐκατέρα
καὶ τὴν λοιπὴν γωνίαν
τῇ λοιπῇ γωνίᾳ·
ἴση ἄρα
ἡ μὲν ΑΒ πλευρὰ τῇ ΓΔ,
ἡ δὲ ΑΓ τῇ ΒΔ,
καὶ ἔτι ἴση ἐστὶν ἡ ὑπὸ ΒΑΓ γωνία
τῇ ὑπὸ ΓΔΒ.
καὶ ἐπεὶ ἴση ἐστὶν ἡ μὲν ὑπὸ ΑΒΓ γωνία
τῇ ὑπὸ ΒΓΔ,
ἡ δὲ ὑπὸ ΓΒΔ τῇ ὑπὸ ΑΓΒ,
ὅλη ἄρα ἡ ὑπὸ ΑΒΔ
ὅλη τῇ ὑπὸ ΑΓΔ
ἐστὶν ἴση.
ἐδείχθη δὲ καὶ
ἡ ὑπὸ ΒΑΓ τῇ ὑπὸ ΓΔΒ ἴση.

Τῶν ἄρα παραλληλογράμμων χωρίων
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσίν.

Paralelkenar alanların,
karşit kenar ve açıları
eşittir birbirine,
ve köşegen onları ikiye böler.

Verilmiş olsun
bir paralelkenar alan
ΑΓΔΒ;
ve onun bir köşegeni, ΒΓ.

iddia ediyorum ki
ΑΓΔΒ paralelkenarının
karşit kenar ve açıları
eşittir birbirine,
ve ΒΓ köşegeni onu ikiye böler.

Çünkü, paralel olduğundan
ΑΒ, ΓΔ doğrusuna,
ve bunların üzerine düştüğünden
bir ΒΓ doğrusu,
ters ΑΒΓ ve ΒΓΔ açıları
eşittir birbirlerine.
Dahası, paralel olduğundan
ΑΓ, ΒΔ doğrusuna,
ve bunların üzerine düştüğünden
ΒΓ,
ters açılar ΑΓΒ ve ΓΒΔ
eşittir birbirlerine.
Şimdi iki üçgen vardır;
ΑΒΓ ve ΒΓΔ,
iki ΑΒΓ ve ΒΓΑ açıları
iki ΒΓΔ ve ΓΒΔ açılara
eşit olan,
her biri birine,
ve bir kenarı, bir kenarına eşit olan,
eşit açılarının yanında olan,
onların ortak ΒΓ kenarı;
o zaman kalan kenarları da
kalan kenarlarına
eşit olacaklar,
her biri birine,
ve kalan açı
kalan açiya;
eşit, dolayısıyla,
ΑΒ kenarı ΓΔ kenarına,
ve ΑΓ, ΒΔ kenarına,
ve eşittir ΒΑΓ açısı
ΓΔΒ açısına.
Ve eşit olduğundan ΑΒΓ,
ΒΓΔ açısına,
ve ΓΒΔ, ΑΓΒ açısına,
dolayısıyla açının tamamı ΑΒΔ,
açının tamamına, ΑΓΔ
eşittir.
Ve gösterilmişti ayrıca
ΒΑΓ ile ΓΔΒ açısının eşitliği.

Dolayısıyla, paralelkenar alanların,
karşit kenar ve açıları
eşittir birbirlerine.

I say then that
also the diameter them cuts in two.

For, since equal is AB to $\Gamma\Delta$,
and common [is] B Γ ,
the two AB and B Γ
to the two $\Gamma\Delta$ and B Γ
equal are,
either to either;
and angle AB Γ
to angle B $\Gamma\Delta$
equal.

Therefore also the base A Γ
to the base ΔB
equal.

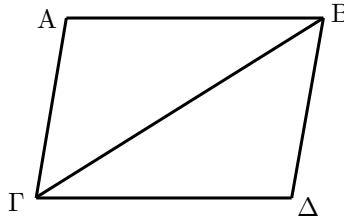
Therefore also the AB Γ triangle
to the B $\Gamma\Delta$ triangle
is equal.

Therefore the B Γ diameter cuts in two
the AB $\Gamma\Delta$ parallelogram;
—just what it was necessary to show.

Λέγω δὴ, ὅτι
καὶ ἡ διάμετρος αὐτὰ διχα τέμνει.

ἐπεὶ γὰρ ἴση ἐστὶν ἡ AB τῆς $\Gamma\Delta$,
κοινὴ δὲ ἡ B Γ ,
δύο δὴ αἱ AB, B Γ
δυοὶ ταῖς $\Gamma\Delta$, B Γ
ἴσαι εἰσὶν
ἐκατέρωθεν ἐκατέρωθεν
καὶ γωνία ἡ ὑπὸ AB Γ
γωνία τῆς ὑπὸ B $\Gamma\Delta$
ἴση.
καὶ βᾶσις ἄρα ἡ A Γ
τῆς ΔB
ἴση.
καὶ τὸ AB Γ [ἄρα] τρίγωνον
τῷ B $\Gamma\Delta$ τριγώνῳ
ἴσον ἐστίν.

Ἡ ἄρα B Γ διάμετρος διχα τέμνει
τὸ AB $\Gamma\Delta$ παραλληλόγραμμον·
ὅπερ ἔδει δεῖξαι.



Şimdi iddia ediyorum ki
köşegen de onları ikiye keser.

Çünkü, eşit olduğundan AB, $\Gamma\Delta$ kenarına,
ve B Γ ortak,
AB ve B Γ ikilisi
- $\Gamma\Delta$ ve B Γ ikilisine
eşittirler,
her biri birine;
ve AB Γ açısı
B $\Gamma\Delta$ açısına
eşittir.
Dolayısıyla A Γ tabanı da
 ΔB tabanına
eşittir.
Dolayısıyla AB Γ üçgeni de
B $\Gamma\Delta$ üçgenine
eşittir.

Dolayısıyla B Γ köşegeni ikiye böler
AB $\Gamma\Delta$ paralelkenarını;
—gösterilmesi gereken tam buydu.

1.35

Parallelograms
on the same base being
and in the same parallels
equal to one another are.

Let there be
parallelograms
AB $\Gamma\Delta$ and EB $\Gamma\Delta$
on the same base, B Γ ,
and in the same parallels,
AZ and B Γ .

I say that
equal is
AB $\Gamma\Delta$
to the parallelogram EB $\Gamma\Delta$.

For, since
a parallelogram is AB $\Gamma\Delta$,
equal is A Δ to B Γ .
Similarly then also,
EZ to B Γ is equal;
so that also A Δ to EZ is equal;
and common [is] ΔE ;
therefore AE, as a whole,
to ΔZ , as a whole,
is equal.

Τὰ παραλληλόγραμμα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
παραλληλόγραμμα
τὰ AB $\Gamma\Delta$, EB $\Gamma\Delta$
ἐπὶ τῆς αὐτῆς βάσεως τῆς B Γ
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς AZ, B Γ .

λέγω, ὅτι
ἴσον ἐστὶ
τὸ AB $\Gamma\Delta$
τῷ EB $\Gamma\Delta$ παραλληλόγραμμῳ.

Ἐπεὶ γὰρ
παραλληλόγραμμὸν ἐστὶ τὸ AB $\Gamma\Delta$,
ἴση ἐστὶν ἡ A Δ τῆς B Γ .
διὰ τὰ αὐτὰ δὴ καὶ
ἡ EZ τῆς B Γ ἐστὶν ἴση·
ὥστε καὶ ἡ A Δ τῆς EZ ἐστὶν ἴση·
καὶ κοινὴ ἡ ΔE .
ὅλη ἄρα ἡ AE
ὅλη τῆς ΔZ
ἐστὶν ἴση.

Paralelkenarlar;
aynı tabanda olan
ve aynı paralellerde olanlar,
birbirlerine eşittir.

Verilmiş olsun
paralelkenarlar,
AB $\Gamma\Delta$ ve EB $\Gamma\Delta$,
aynı B Γ tabanında,
ve aynı
AZ ve B Γ paralellerinde.

İddia ediyorum ki
eşittir
AB $\Gamma\Delta$
EB $\Gamma\Delta$ paralelkenarına.

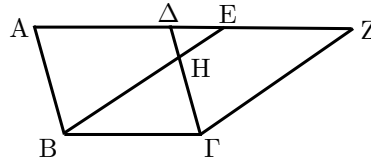
Çünkü
bir paralelkenar olduğundan AB $\Gamma\Delta$,
eşittir A Δ , B Γ kenarına.
Benzer şekilde o zaman,
EZ, B Γ kenarına eşittir;
böylece A Δ da EZ kenarına eşittir;
ve ortaktır ΔE ;
dolayısıyla AE, bir bütün olarak,
 ΔZ kenarına
eşittir.

Is also AB to $\Delta\Gamma$ equal.
 Then the two EA and AB
 to the two Z Δ and $\Delta\Gamma$
 equal are
 either to either;
 also angle Z $\Delta\Gamma$
 to EAB
 is equal,
 the exterior to the interior;
 therefore the base EB
 to the base Z Γ
 is equal,
 and triangle EAB
 to triangle $\Delta Z\Gamma$
 equal will be;
 suppose has been removed, commonly,
 ΔHE ;
 therefore the trapezium ABH Δ that
 remains
 to the trapezium EHGZ that remains
 is equal;
 let be added in common
 the triangle HBG;
 therefore the parallelogram AB $\Gamma\Delta$ as
 a whole
 to the parallelogram EB ΓZ as a whole
 is equal.

Therefore parallelograms
 on the same base being
 and in the same parallels
 equal to one another are;
 —just what it was necessary to show.

ἔστι δὲ καὶ ἡ AB τῆ $\Delta\Gamma$ ἴση·
 δύο δὲ αἱ EA, AB
 δύο ταῖς Z Δ , $\Delta\Gamma$
 ἴσαι εἰσὶν
 ἑκατέρα ἑκατέρῃ·
 καὶ γωνία ἡ ὑπὸ Z $\Delta\Gamma$
 γωνία τῆ ὑπὸ EAB
 ἔστιν ἴση
 ἢ ἐκτὸς τῆ ἐντὸς·
 βάσις ἄρα ἡ EB
 βάσει τῆ Z Γ
 ἴση ἐστίν,
 καὶ τὸ EAB τρίγωνον
 τῷ $\Delta Z\Gamma$ τριγώνῳ
 ἴσον ἔσται·
 κοινὸν ἀφηρήσθω τὸ ΔHE ·
 λοιπὸν ἄρα τὸ ABH Δ τραπέζιον
 λοιπῷ τῷ EHGZ τραπέζιῳ
 ἔστιν ἴσον·
 κοινὸν προσκείσθω τὸ HBG τρίγωνον·
 ὅλον ἄρα τὸ AB $\Gamma\Delta$ παραλληλόγραμμον
 ὅλῳ τῷ EB ΓZ παραλληλογράμμῳ
 ἴσον ἐστίν.

Τὰ ἄρα παραλληλόγραμμα
 τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
 καὶ ἐν ταῖς αὐταῖς παραλλήλοις
 ἴσα ἀλλήλοις ἐστίν·
 ὅπερ εἶδει δεῖξαι.



AB da $\Delta\Gamma$ kenarına eşittir.
 O zaman EA ve AB ikilisi
 Z Δ ve $\Delta\Gamma$ ikilisine
 eşittirler
 her biri birine;
 ve Z $\Delta\Gamma$ açısı da
 EAB açısına
 eşittir,
 dış açı, iç açıya;
 dolayısıyla EB tabanı
 Z Γ tabanına
 eşittir,
 ve EAB üçgeni
 $\Delta Z\Gamma$ üçgenine
 eşit olacak;
 kaldırılmış olsun, ortak olarak,
 ΔHE ;
 dolayısıyla kalan ABH Δ yamuğu
 kalan EHGZ yamuğuna
 eşittir;
 eklenmiş olsun her ikisine birden
 HBG üçgeni;
 dolayısıyla AB $\Gamma\Delta$ paralelkenarın
 tamamı
 EB ΓZ paralelkenarın tamamına
 eşittir.

Dolayısıyla paralelkenarlar;
 aynı tabanda olan
 ve aynı paralellerde olanlar,
 birbirlerine eşittir;
 —gösterilmesi gereken tam buydu.

1.36

Parallelograms
 that are on equal bases
 and in the same parallels
 are equal to one another.

Let there be
 parallelograms
 AB $\Gamma\Delta$ and EZH Θ
 on equal bases,
 B Γ and ZH,
 and in the same parallels,
 A Θ and BH.

I say that
 equal is
 parallelogram AB $\Gamma\Delta$
 to EZH Θ .

For, suppose have been joined

Τὰ παραλληλόγραμμα
 τὰ ἐπὶ ἴσων βάσεων ὄντα
 καὶ ἐν ταῖς αὐταῖς παραλλήλοις
 ἴσα ἀλλήλοις ἐστίν.

Ἐστω
 παραλληλόγραμμα
 τὰ AB $\Gamma\Delta$, EZH Θ
 ἐπὶ ἴσων βάσεων ὄντα
 τῶν B Γ , ZH
 καὶ ἐν ταῖς αὐταῖς παραλλήλοις
 ταῖς A Θ , BH·

λέγω, ὅτι
 ἴσον ἐστὶ
 τὸ AB $\Gamma\Delta$ παραλληλόγραμμον
 τῷ EZH Θ .

Ἐπεξεύχθωσαν γὰρ

Paralelkenarlar;
 eşit tabanlarda olanlar
 ve aynı paralellerde olanlar
 eşittirler birbirlerine.

Verilmiş olsun
 paralelkenarlar
 AB $\Gamma\Delta$ ve EZH Θ
 eşit
 B Γ ve ZH tabanlarında,
 ve aynı
 A Θ ve BH paralellerinde.

İddia ediyorum ki
 eşittir
 AB $\Gamma\Delta$,
 EZH Θ paralelkenarına.

Çünkü, varsayılın birleştirilmiş

BE and $\Gamma\Theta$.

And since equal are $B\Gamma$ and ZH ,
but ZH to $E\Theta$ is equal,
therefore also $B\Gamma$ to $E\Theta$ is equal.
And [they] are also parallel.
Also EB and $\Theta\Gamma$ join them.
And [STRAIGHTS] that join equals and
parallels in the same parts
are equal and parallel.
[Also therefore EB and $H\Theta$
are equal and parallel.]
Therefore a parallelogram is $EB\Gamma\Theta$.
And it is equal to $AB\Gamma\Delta$.
For it has the same base as it,
 $B\Gamma$,
and in the same parallels
as it is, $B\Gamma$ and $A\Theta$.
For the same [reason] then,
also $EZH\Theta$ to it, [namely] $EB\Gamma\Theta$,
is equal;
so that parallelogram $AB\Gamma\Delta$
to $EZH\Theta$ is equal.

Therefore parallelograms
that are on equal bases
and in the same parallels
are equal to one another;
—just what it was necessary to show.

αί BE , $\Gamma\Theta$.

καὶ ἐπεὶ ἴση ἐστὶν ἡ $B\Gamma$ τῆς ZH ,
ἀλλὰ ἡ ZH τῆς $E\Theta$ ἐστὶν ἴση,
καὶ ἡ $B\Gamma$ ἄρα τῆς $E\Theta$ ἐστὶν ἴση.
εἰσὶ δὲ καὶ παράλληλοι.
καὶ ἐπιζευγνύουσιν αὐτὰς αἱ EB , $\Theta\Gamma$.
αἱ δὲ τὰς ἴσας τε καὶ παραλλήλους ἐπὶ
τὰ αὐτὰ μέρη ἐπιζευγνύουσαι
ἴσαι τε καὶ παράλληλοί εἰσι
[καὶ αἱ EB , $\Theta\Gamma$ ἄρα
ἴσαι τέ εἰσι καὶ παράλληλοι].
παράλληλόγραμμον ἄρα ἐστὶ τὸ $EB\Gamma\Theta$.
καὶ ἐστὶν ἴσον τῷ $AB\Gamma\Delta$.
βάσιν τε γὰρ αὐτῶ τὴν αὐτὴν ἔχει
τὴν $B\Gamma$,
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἐστὶν αὐτῶ ταῖς $B\Gamma$, $A\Theta$.
διὰ τὰ αὐτὰ δὴ
καὶ τὸ $EZH\Theta$ τῷ αὐτῷ τῷ $EB\Gamma\Theta$
ἐστὶν ἴσον.
ὥστε καὶ τὸ $AB\Gamma\Delta$ παράλληλόγραμμον
τῷ $EZH\Theta$ ἐστὶν ἴσον.

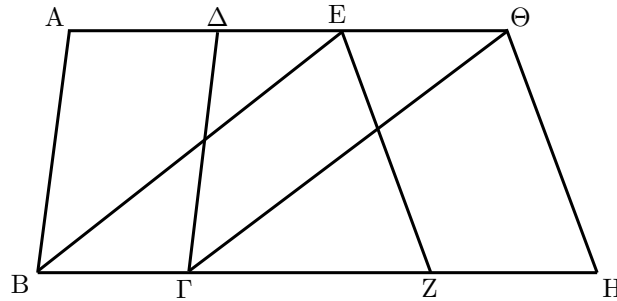
Τὰ ἄρα παράλληλόγραμμα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.
ὅπερ εἶδει δεῖξαι.

olduğu

BE ile $\Gamma\Theta$ kenarlarımın.

Ve eşit olduğundan $B\Gamma$ ile ZH ,
ama ZH , $E\Theta$ kenarına eşittir,
dolayısıyla $B\Gamma$ da $E\Theta$ kenarına eşittir.
Ve paraleldirler de.
Ayrıca EB ve $\Theta\Gamma$ onları birleştirir.
Ve eşit ve paralelleri aynı tarafta bir-
leştiren doğrular
eşit ve paraleldirler.
[Yine dolayısıyla EB ve $H\Theta$
eşit ve paraleldirler.]
Dolayısıyla $EB\Gamma\Theta$ bir paralelkenardır.
Ve eşittir $AB\Gamma\Delta$ paralelkenarına.
Çünkü onunla aynı,
 $B\Gamma$ tabanı vardır,
ve onunla aynı,
 $B\Gamma$ ve $A\Theta$ paralellerindedir.
Aynı sebeple o şimdi,
 $EZH\Theta$ da ona, [yani] $EB\Gamma\Theta$ paralelke-
narına,
eşittir;
böylece $AB\Gamma\Delta$,
 $EZH\Theta$ paralelkenarına eşittir.

Dolayısıyla paralelkenarlar;
eşit tabanlarda olanlar
ve aynı paralellerde olanlar
eşittirler birbirlerine;
—gösterilmesi gereken tam buydu.



1.37

Triangles
that are on the same base
and in the same parallels
are equal to one another.

Let there be
triangles $AB\Gamma$ and $\Delta B\Gamma$,
on the same base $B\Gamma$
and in the same parallels
 $A\Delta$ and $B\Gamma$.

I say that
equal is
triangle $AB\Gamma$
to triangle $\Delta B\Gamma$.

Suppose has been extended

Τὰ τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
τρίγωνα τὰ $AB\Gamma$, $\Delta B\Gamma$
ἐπὶ τῆς αὐτῆς βάσεως τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς $A\Delta$, $B\Gamma$.

λέγω, ὅτι
ἴσον ἐστὶ
τὸ $AB\Gamma$ τρίγωνον
τῷ $\Delta B\Gamma$ τριγώνῳ.

Ἐχβεβλήσθω

Üçgenler;
aynı tabanda
ve aynı paralellerde olanlar,
eşittir birbirlerine.

Verilmiş olsun
 $AB\Gamma$ ve $\Delta B\Gamma$ üçgenleri,
aynı $B\Gamma$ tabanında
ve aynı
 $A\Delta$ ve $B\Gamma$ paralellerinde.

İddia ediyorum ki
eşittir
 $AB\Gamma$ üçgeni
 $\Delta B\Gamma$ üçgenine.

Varsayılın uzatılmış olduğu

$A\Delta$ on both sides to E and Z ,
and through B ,
parallel to ΓA
has been drawn BE ,
and through Γ
parallel to $B\Delta$
has been drawn ΓZ .

Therefore a parallelogram
is either of $EB\Gamma A$ and $\Delta B\Gamma Z$;
and they are equal;
for they are on the same base,
 $B\Gamma$,
and in the same parallels,
 $B\Gamma$ and EZ ;
and [it] is
of the parallelogram $EB\Gamma A$
half
—the triangle $AB\Gamma$;
for the diameter AB cuts it in two;
and of the parallelogram $\Delta B\Gamma Z$
half
—the triangle $\Delta B\Gamma$;
for the diameter $\Delta\Gamma$ cuts it in two.
[And halves of equals
are equal to one another.]
Therefore equal is
the triangle $AB\Gamma$ to the triangle $\Delta B\Gamma$.

Therefore triangles
that are on the same base
and in the same parallels
are equal to one another;
—just what it was necessary to show.

ἡ $A\Delta$ ἐφ' ἐκάτερα τὰ μέρη ἐπὶ τὰ E , Z ,
καὶ διὰ μὲν τοῦ B
τῆ ΓA παράλληλος
ἤχθω ἡ BE ,
διὰ δὲ τοῦ Γ
τῆ $B\Delta$ παράλληλος
ἤχθω ἡ ΓZ .

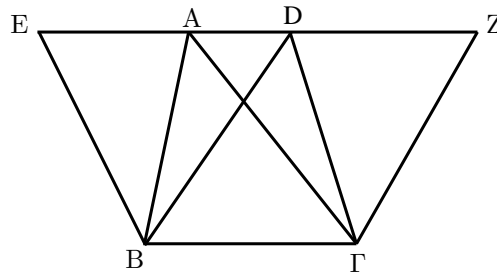
παράλληλόγραμμον ἄρα
ἐστὶν ἐκάτερον τῶν $EB\Gamma A$, $\Delta B\Gamma Z$
καὶ εἰσιν ἴσα·
ἐπὶ τε γὰρ τῆς αὐτῆς βάσεως εἰσὶ
τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς $B\Gamma$, EZ
καὶ ἐστὶ
τοῦ μὲν $EB\Gamma A$ παραλληλογράμμου
ἡμισυ
τὸ $AB\Gamma$ τρίγωνον·
ἡ γὰρ AB διάμετρος αὐτὸ δίχα τέμνει·
τοῦ δὲ $\Delta B\Gamma Z$ παραλληλογράμμου
ἡμισυ
τὸ $\Delta B\Gamma$ τρίγωνον·
ἡ γὰρ $\Delta\Gamma$ διάμετρος αὐτὸ δίχα τέμνει.
[τὰ δὲ τῶν ἴσων ἡμίση
ἴσα ἀλλήλοις ἐστίν].
ἴσον ἄρα ἐστὶ
τὸ $AB\Gamma$ τρίγωνον τῷ $\Delta B\Gamma$ τριγώνῳ.

Τὰ ἄρα τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.

$A\Delta$ doğrusunun her iki kenarda E ve
 Z noktalarına,
ve B noktasından,
 ΓA kenarına paralel
 BE çizilmiş olsun,
ve Γ noktasından
 $B\Delta$ kenarına papalel
 ΓZ çizilmiş olsun.

Dolayısıyla birer paralelkenardır
 $EB\Gamma A$ ile $\Delta B\Gamma Z$;
ve bunlar eşittir;
aynı
 $B\Gamma$ tabanında,
ve aynı,
 $B\Gamma$ ve EZ paralellerinde oldukları için;
ve
 $EB\Gamma A$ paralelkenarının
yarısı
— $AB\Gamma$ üçgenidir;
 AB köşegeni onu ikiye kestiği için;
ve $\Delta B\Gamma Z$ paralelkenarının
yarısı
— $\Delta B\Gamma$ üçgenidir;
 $\Delta\Gamma$ köşegeni onu ikiye kestiği için.
[Ve eşitlerin yaruları
eşittirler birbirlerine.]
Dolayısıyla eşittir
 $AB\Gamma$ üçgeni $\Delta B\Gamma$ üçgenine.

Dolayısıyla üçgenler;
aynı tabanda
ve aynı paralellerde olanlar,
eşittir birbirlerine;
—gösterilmesi gereken tam buydu.



1.38

Triangles
that are on equal bases
and in the same parallels
are equal to one another.

Let there be
triangles $AB\Gamma$ and ΔEZ
on equal bases $B\Gamma$ and EZ
and in the same parallels
 BZ and $A\Delta$.

I say that
equal is
triangle $AB\Gamma$

Τὰ τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
τρίγωνα τὰ $AB\Gamma$, ΔEZ
ἐπὶ ἴσων βάσεων τῶν $B\Gamma$, EZ
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BZ , $A\Delta$.

λέγω, ὅτι
ἴσον ἐστὶ
τὸ $AB\Gamma$ τρίγωνον

Üçgenler;
eşit tabanlarda
ve aynı paralellerde olanlar,
eşittir birbirlerine.

Verilmiş olsun
 $AB\Gamma$ ve ΔEZ üçgenleri
eşit $B\Gamma$ ve EZ tabanlarında
ve aynı
 BZ ve $A\Delta$ paralellerinde.

İddia ediyorum ki
eşittir
 $AB\Gamma$ üçgeni

to triangle ΔEZ .

For, suppose has been extended $A\Delta$ on both sides to H and Θ , and through B , parallel to ΓA , has been drawn BH , and through Z , parallel to ΔE , has been drawn $Z\Theta$.

Therefore a parallelogram is either of $HB\Gamma A$ and $\Delta EZ\Theta$; and $HB\Gamma A$ [is] equal to $\Delta EZ\Theta$; for they are on equal bases, $B\Gamma$ and EZ , and in the same parallels, BZ and $H\Theta$; and [it] is of the parallelogram $HB\Gamma A$ half —the triangle $AB\Gamma$. For the diameter AB cuts it in two; and of the parallelogram $\Delta EZ\Theta$ half —the triangle $ZE\Delta$; for the diameter ΔZ cuts it in two. [And halves of equals are equal to one another.] Therefore equal is the triangle $AB\Gamma$ to the triangle ΔEZ .

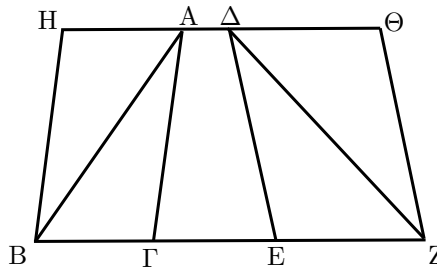
Therefore triangles that are on equal bases and in the same parallels are equal to one another; —just what it was necessary to show.

τῷ ΔEZ τριγώνῳ.

Ἐκβεβλήσθω γὰρ ἡ $A\Delta$ ἐφ' ἐκάτερα τὰ μέρη ἐπὶ τὰ H , Θ , καὶ διὰ μὲν τοῦ B τῇ ΓA παράλληλος ἦχθω ἡ BH , διὰ δὲ τοῦ Z τῇ ΔE παράλληλος ἦχθω ἡ $Z\Theta$.

παράλληλόγραμμον ἄρα ἐστὶν ἐκάτερον τῶν $HB\Gamma A$, $\Delta EZ\Theta$ · καὶ ἴσον τὸ $HB\Gamma A$ τῷ $\Delta EZ\Theta$ · ἐπὶ τε γὰρ ἴσων βάσεων εἰσι τῶν $B\Gamma$, EZ καὶ ἐν ταῖς αὐταῖς παραλλήλοις ταῖς BZ , $H\Theta$ · καὶ ἐστὶ τοῦ μὲν $HB\Gamma A$ παραλληλογράμμου ἡμισυ τὸ $AB\Gamma$ τρίγωνον· ἡ γὰρ AB διάμετρος αὐτὸ δίχα τέμνει· τοῦ δὲ $\Delta EZ\Theta$ παραλληλογράμμου ἡμισυ τὸ $ZE\Delta$ τρίγωνον· ἡ γὰρ ΔZ διάμετρος αὐτὸ δίχα τέμνει [τὰ δὲ τῶν ἴσων ἡμίση ἴσα ἀλλήλοις ἐστίν]· ἴσον ἄρα ἐστὶ τὸ $AB\Gamma$ τρίγωνον τῷ ΔEZ τριγώνῳ.

Τὰ ἄρα τρίγωνα τὰ ἐπὶ ἴσων βάσεων ὄντα καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἴσα ἀλλήλοις ἐστίν· ὅπερ ἔδει δεῖξαι.



1.39

Equal triangles that are on the same base and in the same parts are also in the same parallels.

Let there be equal triangles $AB\Gamma$ and $\Delta B\Gamma$, being on the same base and on the same side of $B\Gamma$.

Τὰ ἴσα τρίγωνα τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα καὶ ἐπὶ τὰ αὐτὰ μέρη καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐστω ἴσα τρίγωνα τὰ $AB\Gamma$, $\Delta B\Gamma$ ἐπὶ τῆς αὐτῆς βάσεως ὄντα καὶ ἐπὶ τὰ αὐτὰ μέρη τῆς $B\Gamma$.

ΔEZ üçgenine.

Çünkü varsayalım uzatılmış olduğu $A\Delta$ kenarının her iki kenarda H ve Θ noktalarına, ve B noktasından, ΓA kenarına paralel, BH çizilmiş olsun, ve Z noktasından, ΔE kenarına paralel, $Z\Theta$ çizilmiş olsun.

Dolayısıyla birer paralelkenardır $HB\Gamma A$ ile $\Delta EZ\Theta$; ve $HB\Gamma A$ eşittir $\Delta EZ\Theta$ paralelkenarına; eşit, $B\Gamma$ ve EZ tabanlarında, ve aynı, BZ ve $H\Theta$ paralellerinde oldukları için; ve $HB\Gamma A$ paralelkenarının yarısı — $AB\Gamma$ üçgenidir. AB köşegeni onu ikiye kestiği için; ve $\Delta EZ\Theta$ paralelkenarının yarısı — $ZE\Delta$ üçgenidir; ΔZ köşegeni onu ikiye kestiği için. [Ve eşitlerin yarıları eşittirler birbirlerine.] Dolayısıyla eşittir $AB\Gamma$ üçgeni ΔEZ üçgenine.

Dolayısıyla üçgenler; eşit tabanlarda ve aynı paralellerde olanlar, eşittir birbirlerine; —gösterilmesi gereken tam buydu.

Eşit üçgenler; aynı tabanda ve onun aynı tarafında olan, aynı paralellerdedirler de.

Verilmiş olsun $AB\Gamma$ ve $\Delta B\Gamma$ eşit üçgenleri, aynı $B\Gamma$ tabanında ve onun aynı tarafında olan .

I say that
they are also in the same parallels.

For suppose has been joined $A\Delta$.

I say that
parallel is $A\Delta$ to $B\Gamma$.

For if not,
suppose there has been drawn
through the point A
parallel to the STRAIGHT $B\Gamma$
 AE ,
and there has been joined EF .
Equal therefore is
the triangle $AB\Gamma$
to the triangle $EB\Gamma$;
for on the same base
as it is, $B\Gamma$,
and in the same parallels.
But $AB\Gamma$ is equal to $\Delta B\Gamma$.
Also therefore $\Delta B\Gamma$ to $EB\Gamma$ is equal,
the greater to the less;
which is impossible.
Therefore is not parallel AE to $B\Gamma$.
Similarly then we shall show that
neither is any other but $A\Delta$;
therefore $A\Delta$ is parallel to $B\Gamma$.

Therefore equal triangles
that are on the same base
and in the same parts
are also in the same parallels;
—just what it was necessary to show.

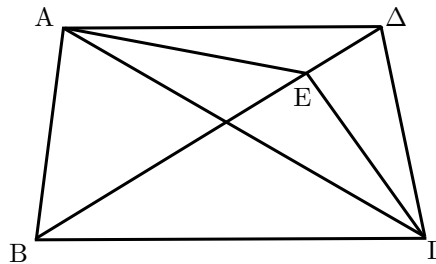
λέγω, ὅτι
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐπεζεύχθω γὰρ ἡ $A\Delta$.

λέγω, ὅτι
παραλληλός ἐστιν ἡ $A\Delta$ τῇ $B\Gamma$.

Εἰ γὰρ μή,
ῥηθῶ
διὰ τοῦ A σημείου
τῇ $B\Gamma$ εὐθείᾳ παράλληλος
ἡ AE ,
καὶ ἐπεζεύχθω ἡ EF .
ἴσον ἄρα ἐστὶ
τὸ $AB\Gamma$ τρίγωνον
τῷ $EB\Gamma$ τριγώνῳ·
ἐπὶ τε γὰρ τῆς αὐτῆς βάσεως
ἐστὶν αὐτῷ τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις.
ἀλλὰ τὸ $AB\Gamma$ τῷ $\Delta B\Gamma$ ἐστὶν ἴσον·
καὶ τὸ $\Delta B\Gamma$ ἄρα τῷ $EB\Gamma$ ἴσον ἐστὶ
τὸ μείζον τῷ ἐλάσσονι·
ὅπερ ἐστὶν ἀδύνατον·
οὐκ ἄρα παράλληλος ἐστὶν ἡ AE τῇ $B\Gamma$.
ὁμοίως δὲ δείξομεν, ὅτι
οὐδ' ἄλλη τις πλὴν τῆς $A\Delta$
ἡ $A\Delta$ ἄρα τῇ $B\Gamma$ ἐστὶ παράλληλος.

Τὰ ἄρα ἴσα τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.



İddia ediyorum ki
aynı paralellerdedirler de.

Çünkü $A\Delta$ doğrusunun birleştirilmiş
olduğu varsayılsın.

İddia ediyorum ki
paraleldir $A\Delta$, $B\Gamma$ tabanına.

Çünkü eğer değil ise,
çizilmiş olduğu varsayılsın
A noktasından
 $B\Gamma$ doğrusuna paralel
 AE doğrusunun,
ve birleştirildiği EF doğrusunun.
Eşittir dolayısıyla
 $AB\Gamma$ üçgeni
 $EB\Gamma$ üçgenine;
onunla aynı
 $B\Gamma$ tabanında,
ve aynı paralellerde olduğu için.
Ama $AB\Gamma$ eşittir $\Delta B\Gamma$ üçgenine.
Ve dolayısıyla $\Delta B\Gamma$, $EB\Gamma$ üçgenine
eşittir,
büyük küçüğe;
ki bu imkansızdır.
Dolayısıyla paralel değildir AE , $B\Gamma$
doğrusuna.
Benzer şekilde o zaman göstereceğiz ki
 $A\Delta$ dışındakiler de paralel değildirdi ;
dolayısıyla $A\Delta$, $B\Gamma$ doğrusuna paar-
aleldir.

Dolayısıyla eşit üçgenler;
aynı tabanda
ve onun aynı tarafında olan,
aynı paralellerdedirler de;
—gösterilmesi gereken tam buydu.

1.40

Equal triangles
that are on equal bases
and in the same parts
are also in the same parallels.

Let there be
equal triangles $AB\Gamma$ and $\Gamma\Delta E$,
on equal bases $B\Gamma$ and ΓE ,
and in the same parts.

Τὰ ἴσα τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐστω
ἴσα τρίγωνα τὰ $AB\Gamma$, $\Gamma\Delta E$
ἐπὶ ἴσων βάσεων τῶν $B\Gamma$, ΓE
καὶ ἐπὶ τὰ αὐτὰ μέρη.

Eşit üçgenler,
eşit tabanlarda
ve aynı tarafta olan,
aynı paralellerdedirler de.

Verilmiş olsun
eşit $AB\Gamma$ ve $\Gamma\Delta E$ üçgenleri,
eşit $B\Gamma$ ve ΓE tabanlarında,
ve aynı tarafta olan.

I say that
they are also in the same parallels.

For suppose $A\Delta$ has been joined.

I say that
parallel is $A\Delta$ to BE .

For if not,
suppose there has been drawn
through the point A ,
parallel to BE ,
 AZ ,

and there has been joined ZE .

Equal therefore is
the triangle $AB\Gamma$
to the triangle $Z\Gamma E$;
for they are on equal bases,
 $B\Gamma$ and ΓE ,

and in the same parallels,
 BE and AZ .

But the triangle $AB\Gamma$
is equal to the [triangle] $\Delta\Gamma E$;
also therefore the [triangle] $\Delta\Gamma E$
is equal to the triangle $Z\Gamma E$,
the greater to the less;
which is impossible.

Therefore is not parallel AZ to BE .

Similarly then we shall show that
neither is any other but $A\Delta$;
therefore $A\Delta$ to BE is parallel.

Therefore equal triangles
that are on equal bases
and in the same parts
are also in the same parallels;
—just what it was necessary to show.

λέγω, ὅτι
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐπεξεύχθω γὰρ ἡ $A\Delta$.

λέγω, ὅτι
παράλληλός ἐστιν ἡ $A\Delta$ τῇ BE .

Εἰ γὰρ μή,
ἦχθω
διὰ τοῦ A
τῇ BE παράλληλος
ἡ AZ ,
καὶ ἐπεξεύχθω ἡ ZE .
ἴσον ἄρα ἐστὶ
τὸ $AB\Gamma$ τρίγωνον
τῷ $Z\Gamma E$ τριγώνῳ·
ἐπὶ τε γὰρ ἴσων βάσεων εἰσι
τῶν $B\Gamma$, ΓE
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BE , AZ .

ἀλλὰ τὸ $AB\Gamma$ τρίγωνον
ἴσον ἐστὶ τῷ $\Delta\Gamma E$ [τρίγωνῳ].
καὶ τὸ $\Delta\Gamma E$ ἄρα [τρίγωνον]
ἴσον ἐστὶ τῷ $Z\Gamma E$ τριγώνῳ
τὸ μείζον τῷ ἐλάσσονι·
ὅπερ ἐστὶν ἀδύνατον·
οὐκ ἄρα παράλληλος ἡ AZ τῇ BE .
ὁμοίως δὲ δεῖξομεν, ὅτι
οὐδ' ἄλλη τις πλὴν τῆς $A\Delta$
ἡ $A\Delta$ ἄρα τῇ BE ἐστὶ παράλληλος.

Τὰ ἄρα ἴσα τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.

İddia ediyorum ki
aynı paralellerdedirler de.

Çünkü varsayalım $A\Delta$ doğrusunun
birleştirildiği.

İddia ediyorum ki
paraleldir $A\Delta$, BE doğrusuna.

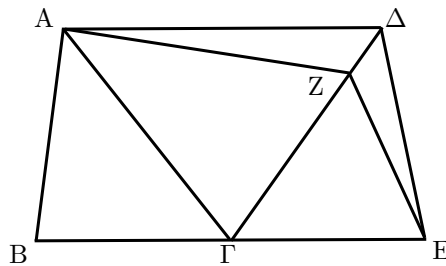
Çünkü eğer değil ise,
varsayalım birleştirildiği
 A noktasından,
 BE doğrusuna paralel,
 AZ doğrusunun,
ve birleştirildiği ZE doğrusunun.

Dolayısıyla eşittir
 $AB\Gamma$ üçgeni
 $Z\Gamma E$ üçgenine;
eşit,
 $B\Gamma$ ve ΓE tabanlarında,
ve aynı,
 BE ve AZ paralellerinde oldukları için.

Fakat $AB\Gamma$ üçgeni
eşittir $\Delta\Gamma E$ üçgenine;
ve dolayısıyla $\Delta\Gamma E$ üçgenini
eşittir $Z\Gamma E$ üçgenine,
büyük küçüğe;
ki bu imkansızdır.
Dolayısıyla paralel değildir AZ , BE
doğrusuna.

Benzer şekilde o zaman göstereceğiz ki
 $A\Delta$ dışındakiler de paralel değildir;
dolayısıyla $A\Delta$, BE doğrusuna par-
aleldir.

Dolayısıyla eşit üçgenler,
eşit tabanlarda
ve aynı tarafta olan,
aynı paralellerdedirler de;
—gösterilmesi gereken tam buydu.



1.41

If a parallelogram
have the same base as a triangle,
and be in the same parallels,
double is
the parallelogram of the triangle.

For, the parallelogram $AB\Gamma\Delta$
as the triangle $EB\Gamma$,
—suppose it has the same base, $B\Gamma$,

Ἐὰν παραλληλόγραμμον
τριγώνῳ βάσιν τε ἔχη τὴν αὐτὴν
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἦ,
διπλάσιόν ἐστὶ
τὸ παραλληλόγραμμον τοῦ τριγώνου.

Παραλληλόγραμμον γὰρ τὸ $AB\Gamma\Delta$
τριγώνῳ τῷ $EB\Gamma$
βάσιν τε ἐχέτω τὴν αὐτὴν τὴν $B\Gamma$

Eğer bir paralelkenar
bir üçgenle aynı tabana sahipse,
ve aynı paralellerdeyse,
iki katıdır
paralelkenar, üçgenin.

Çünkü $AB\Gamma\Delta$ paralelkenarının
 $EB\Gamma$ üçgeniyle,
—aynı $B\Gamma$ tabanı olduğu varsayalım,

and is in the same parallels,
BF and AE.

I say that
double is
the parallelogram ABΓΔ
of the triangle BEΓ.

For, suppose AΓ has been joined.

Equal is the triangle ABΓ
to the triangle EBΓ;
for it is on the same base as it,
BΓ,
and in the same parallels,
BΓ and AE.
But the parallelogram ABΓΔ
is double of the triangle ABΓ;
for the diameter AΓ cuts it in two;
so that the parallelogram ABΓΔ
also of the triangle EBΓ is double.

Therefore, if a parallelogram
have the same base as a triangle,
and be in the same parallels,
double is
the parallelogram of the triangle;
—just what it was necessary to show.

καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἔστω
ταῖς BΓ, AE·

λέγω, ὅτι
διπλάσιόν ἐστι
τὸ ABΓΔ παραλληλόγραμμον
τοῦ BEΓ τριγώνου.

Ἐπεζεύχθω γὰρ ἡ AΓ.

ἴσον δὴ ἐστὶ τὸ ABΓ τρίγωνον
τῷ EBΓ τριγώνῳ·
ἐπὶ τε γὰρ τῆς αὐτῆς βάσεως ἐστὶν αὐτῷ
τῆς BΓ
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BΓ, AE.
ἀλλὰ τὸ ABΓΔ παραλληλόγραμμον
διπλάσιόν ἐστι τοῦ ABΓ τριγώνου·
ἢ γὰρ AΓ διάμετρος αὐτὸ δίχα τέμνει·
ὥστε τὸ ABΓΔ παραλληλόγραμμον
καὶ τοῦ EBΓ τριγώνου ἐστὶ διπλάσιον.

Ἐὰν ἄρα παραλληλόγραμμον
τριγώνῳ βάσιν τε ἔχη τὴν αὐτὴν
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ᾗ,
διπλάσιόν ἐστὶ
τὸ παραλληλόγραμμον τοῦ τριγώνου·
ὅπερ εἶδει δεῖξαι.

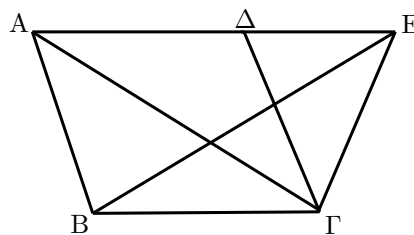
ve aynı
BΓ ve AE paralelerinde oldukları.

İddia ediyorum ki
iki katıdır
ABΓΔ paralelkenarı
BEΓ üçgeninin.

Çünkü, varsayalım AΓ doğrusunun
birleştirildiği.

Eşittir ABΓ üçgeni
EBΓ üçgenine;
onunla aynı,
BΓ tabanına sahip,
ve aynı
BΓ ve AE paralelerinde olduğu için.
Fakat ABΓΔ paralelkenarı
iki katıdır ABΓ üçgeninin;
AΓ köşegeni onu ikiye kestğinden;
böylece ABΓΔ paralelkenarı da
EBΓ üçgeninin iki katıdır.

Dolayısıyla, eğer bir paralelkenar
bir üçgenle aynı tabana sahipse,
ve aynı paralelerdeyse,
iki katıdır
paralelkenar, üçgenin;
—gösterilmesi gereken tam buydu.



1.42

To the given triangle equal,
a parallelogram to construct
in the given rectilinear angle.

Let be
the given triangle ABΓ,
and the given rectilinear angle, Δ.

It is necessary then
to the triangle ABΓ equal
a parallelogram to construct
in the rectilinear angle Δ.

Suppose BΓ has been cut in two at E,
and there has been joined AE,
and there has been constructed
on the STRAIGHT EF,
and at the point E on it,
to angle Δ equal,
FEZ,
also, through A, parallel to EF,

Τῷ δοθέντι τριγώνῳ ἴσον
παραλληλόγραμμον συστήσασθαι
ἐν τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ.

Ἐστω
τὸ μὲν δοθὲν τρίγωνον τὸ ABΓ,
ἢ δὲ δοθεῖσα γωνία εὐθύγραμμος ἡ Δ·

δεῖ δὴ
τῷ ABΓ τριγώνῳ ἴσον
παραλληλόγραμμον συστήσασθαι
ἐν τῇ Δ γωνίᾳ εὐθύγραμμῳ.

Τετμήσθω ἡ BΓ δίχα κατὰ τὸ E,
καὶ ἐπεζεύχθω ἡ AE,
καὶ συνεστάτω
πρὸς τῇ EF εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ E
τῇ Δ γωνίᾳ ἴση
ἢ ὑπὸ FEZ,
καὶ διὰ μὲν τοῦ A τῇ EF παράλληλος

Verilen bir üçgene eşit,
bir paralelkenarı
verilen bir düzkenar açıda inşa etmek.

Verilen
üçgen ABΓ,
ve verilen düzkenar açısı Δ olsun.

Şimdi gerklidir
ABΓ üçgenine eşit
bir paralelkenarın
Δ düzkenar açısına inşa edilmesi.

Varsayalım BΓ kenarının E nok-
tasında ikiye kesildiği
ve AE doğrusunun birleştirildiği,
ve inşa edildiği
EF doğrusunda,
ve üzerindeki E noktasında,
Δ açısına eşit,
FEZ açısının,

suppose AH has been drawn,
and through Γ , parallel to EZ,
suppose GH has been drawn;
therefore a parallelogram is ZEGH.

And since equal is BE to EG,
equal is also
triangle ABE to triangle AEG;
for they are on equal bases,
BE and EG,
and in the same parallels,
BG and AH;
double therefore is
triangle ABG of triangle AEG.
also is
parallelogram ZEGH
double of triangle AEG;
for it has the same base as it,
and
is in the same parallels as it;
therefore is equal
the parallelogram ZEGH
to the triangle ABG.
And it has angle GEZ
equal to the given Δ .

Therefore, to the given triangle ABG
equal,
a parallelogram has been constructed,
ZEGH,
in the angle GEZ,
which is equal to Δ ;
—just what it was necessary to do.

ἤχθω ἡ AH,
διὰ δὲ τοῦ Γ τῆ EZ παράλληλος
ἤχθω ἡ GH·
παράλληλόγραμμον ἄρα ἐστὶ τὸ ZEGH.

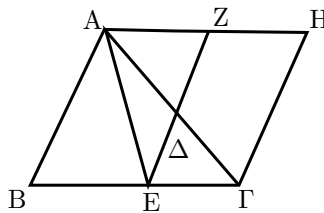
καὶ ἐπεὶ ἴση ἐστὶν ἡ BE τῆ EG,
ἴσον ἐστὶ καὶ
τὸ ABE τρίγωνον τῷ AEG τριγώνω·
ἐπὶ τε γὰρ ἴσων βάσεων εἰσι
τῶν BE, EG
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BG, AH·
διπλάσιον ἄρα ἐστὶ
τὸ ABG τρίγωνον τοῦ AEG τριγώνου.
ἔστι δὲ καὶ
τὸ ZEGH παράλληλόγραμμον
διπλάσιον τοῦ AEG τριγώνου·
βάσιν τε γὰρ αὐτῶ τὴν αὐτὴν ἔχει
καὶ
ἐν ταῖς αὐταῖς ἐστὶν αὐτῶ παραλλήλοις·
ἴσον ἄρα ἐστὶ
τὸ ZEGH παράλληλόγραμμον
τῷ ABG τριγώνω.
καὶ ἔχει τὴν ὑπὸ GEZ γωνίαν
ἴσην τῆ δοθείσῃ τῆ Δ .

Τῷ ἄρα δοθέντι τριγώνω τῷ ABG
ἴσον
παράλληλόγραμμον συνέσταται
τὸ ZEGH
ἐν γωνίᾳ τῆ ὑπὸ GEZ,
ἣτις ἐστὶν ἴση τῆ Δ ·
ὅπερ ἔδει ποιῆσαι.

ayrıca, A noktasından, EG doğrusuna
paralel,
AH doğrusunun çizilmiş olduğu
varsayılınsın,
ve Γ noktasından, EZ doğrusuna par-
alel,
GH doğrusunun çizilmiş olduğu
varsayılınsın;
dolayısıyla ZEGH bir paralelkenardır.

Ve eşit olduğundan BE, EG
doğrusuna,
eşittir
ABE üçgeni de AEG üçgenine;
tabanları
BE ve EG eşit,
ve aynı
BG ve AH paralelerinde oldukları için;
iki katıdır dolayısıyla
ABG üçgeni AEG üçgeninin,
ayrıca
ZEGH paralelkenarı
iki katıdır AEG üçgeninin;
onunla aynı tabanı olduğu,
ve
onunla aynı paralellerde olduğu için;
dolayısıyla eşittir
ZEGH paralelkenarı
ABG üçgenine.
Ve onun GEZ açısı
eşittir verilen Δ açısına.

Dolayısıyla, verilen ABG üçgenine
eşit,
bir paralelkenar,
ZEGH, inşa edilmiş oldu
GEZ açısında,
 Δ açısına eşit olan;
—yapılması gereken tam buydu.



1.43

Of any parallelogram,
of the parallelograms about the diam-
eter,
the complements
are equal to one another.

Let there be
a parallelogram ABGΔ,
and its diameter, AG,
and about AG
let be parallelograms,
EΘ and ZH,¹

Παντὸς παραλληλογράμμου
τῶν περὶ τὴν διάμετρον παραλληλο-
γράμμων
τὰ παραπληρώματα
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
παράλληλόγραμμον τὸ ABGΔ,
διάμετρος δὲ αὐτοῦ ἡ AG,
περὶ δὲ τὴν AG
παράλληλόγραμμα μὲν ἔστω
τὰ EΘ, ZH,

Herhangi bir paralelkenarın,
köşegeni etrafındaki
paralelkenarların,
tümleyenleri
eşittir birbirlerine.

Verilmiş olsun
bir ABGΔ paralelkenarı,
ve onun AG köşegeni,
ve AG etrafında
paralelkenarlar,
EΘ ve ZH,

and the so-called² complements,
BK and KΔ.

I say that
equal is the complement BK
to the complement KΔ.

For, since a parallelogram is
ABΓΔ,
and its diameter, ΑΓ,
equal is
triangle ABΓ to triangle ΑΓΔ.
Moreover, since a parallelogram is
EΘ,
and its diameter, AK,
equal is
triangle AEK to triangle AΘK.
Then for the same [reasons] also
triangle KZΓ to KHΓ is equal.
Since then triangle AEK
is equal to triangle AΘK,
and KZΓ to KHΓ,
triangle AEK with KHΓ
is equal
to triangle AΘK with KZΓ;
also is triangle ABΓ, as a whole,
equal to AΔΓ, as a whole;
therefore the complement BK remain-
ing
to the complement KΔ remaining
is equal.

Therefore, of any parallelogram area,
of the about-the-diameter
parallelograms,
the complements
are equal to one another;
—just what it was necessary to show.

τὰ δὲ λεγόμενα παραπληρώματα
τὰ BK, KΔ.

λέγω, ὅτι
ἴσον ἐστὶ τὸ BK παραπλήρωμα
τῷ KΔ παραπλήρωματι.

Ἐπεὶ γὰρ παραλληλόγραμμὸν ἐστὶ
τὸ ABΓΔ,
διάμετρος δὲ αὐτοῦ ἡ ΑΓ,
ἴσον ἐστὶ
τὸ ABΓ τρίγωνον τῷ ΑΓΔ τριγώνῳ.
πάλιν, ἐπεὶ παραλληλόγραμμὸν ἐστὶ
τὸ EΘ,
διάμετρος δὲ αὐτοῦ ἐστὶν ἡ AK,
ἴσον ἐστὶ
τὸ AEK τρίγωνον τῷ AΘK τριγώνῳ.
διὰ τὰ αὐτὰ δὴ καὶ
τὸ KZΓ τρίγωνον τῷ KHΓ ἐστὶν ἴσον.
ἐπεὶ οὖν τὸ μὲν AEK τρίγωνον
τῷ AΘK τριγώνῳ ἐστὶν ἴσον,
τὸ δὲ KZΓ τῷ KHΓ,
τὸ AEK τρίγωνον μετὰ τοῦ KHΓ
ἴσον ἐστὶ
τῷ AΘK τριγώνῳ μετὰ τοῦ KZΓ.
ἔστι δὲ καὶ ὅλον τὸ ABΓ τρίγωνον
ὅλῳ τῷ AΔΓ ἴσον.
λοιπὸν ἄρα τὸ BK παραπλήρωμα
λοιπῷ τῷ KΔ παραπλήρωματι
ἐστὶν ἴσον.

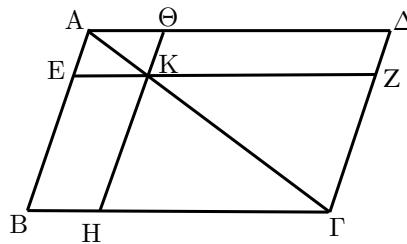
Παντὸς ἄρα παραλληλογράμμου χωρίου
τῶν περὶ τὴν διάμετρον
παραλληλογράμμων
τὰ παραπληρώματα
ἴσα ἀλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.

ve bunların tümleyenleri,
BK ile KΔ.

İddia ediyorum
eşittir BK tümleyeni
KΔ tümleyenine.

Çünkü, bir paralelkenar olduğundan
ABΓΔ,
ve ΑΓ, onun köşegeni,
eşittir
ABΓ üçgeni ΑΓΔ üçgenine.
Dahası, bir paralelkenar olduğundan
EΘ,
AK, onun köşegeni,
eşittir
AEK üçgeni AΘK üçgenine.
Şimdi aynı nedenle
KZΓ eşittir KHΓ üçgenine.
O zaman AEK
eşit olduğundan AΘK üçgenine,
ve KZΓ, KHΓ üçgenine,
AEK ile KHΓ üçgenleri
eşittir
AΘK ile KZΓ üçgenlerine;
ayrıca ABΓ üçgeninin tümü
eşittir AΔΓ üçgeninin tümüne;
dolayısıyla geriye kalan BK tümleyeni,
geriye kalan KΔ tümleyenine
eşittir.

Dolayısıyla, herhangi bir paralelke-
narın,
köşegeni etrafındaki
paralelkenarların,
tümleyenleri
eşittir birbirlerine;
—gösterilmesi gereken tam buydu.



1.44

Along the given STRAIGHT,
equal to the given triangle,
to apply a parallelogram
in the given rectilinear angle.

Let be
the given STRAIGHT AB,

Παρά τὴν δοθεῖσαν εὐθείαν
τῷ δοθέντι τριγώνῳ ἴσον
παραλληλόγραμμον παραβαλεῖν
ἐν τῇ δοθείσῃ γωνίᾳ εὐθυγράμμῳ.

Ἐστω
ἡ μὲν δοθεῖσα εὐθεῖα ἡ AB,

Verilen bir doğru boyunca
verilen bir üçgene eşit,
bir paralel kenarı yerleştirmek
verilen bir düz kenar açıda.

Verilen doğru AB,
ve verilen üçgen Γ,

¹Here Euclid can use two letters without qualification for a parallelogram, because they are not unqualified in the Greek: they take the neuter article, while a line takes the feminine article.

²This is Heath's translation. The Greek does not require any-

thing corresponding to 'so-'. The LSJ lexicon [10] gives the present proposition as the original geometrical use of παραπλήρωμα—other meanings are 'expletive' and a certain flowering herb.

and the given triangle, Γ ,
and the given rectilineal angle, Δ .

It is necessary then
along the given STRAIGHT AB
equal to the given triangle Γ
to apply a parallelogram
in an equal to the angle Δ .

Suppose has been constructed
equal to triangle Γ ,
a parallelogram BEZH
in angle EBH,
which is equal to Δ ;
and let it be laid down
so that on a STRAIGHT is BE
with AB,
and suppose has been drawn through
ZH to Θ ,
and through A,
parallel to either of BH and EZ,
suppose there has been drawn
A Θ ,
and suppose there has been joined
 Θ B.

And since on the parallels A Θ and EZ
fell the STRAIGHT Θ Z,
the angles A Θ Z and Θ ZE
are equal to two RIGHTS.
Therefore B Θ H and HZE
are less than two RIGHTS.
And [STRAIGHTS] from [angles] that
are less
than two RIGHTS,
extended to the infinite,
fall together.
Therefore Θ B and ZE, extended,
fall together.

Suppose they have been extended,
and they have fallen together at K,
and through the point K,
parallel to either of EA and Z Θ ,
suppose has been drawn K Λ ,
and suppose have been extended Θ A
and HB
to the points Λ and M.

A parallelogram therefore is Θ Λ KZ,
a diameter of it is Θ K,
and about Θ K [are]
the parallelograms AH and ME,
and the so-called complements,
 Λ B and BZ;
equal therefore is Λ B to BZ.
But BZ to triangle Γ is equal.
Also therefore Λ B to Γ is equal.
And since equal is
angle HBE to ABM,
but HBE to Δ is equal,
also therefore ABM to Δ

τὸ δὲ δοθὲν τρίγωνον τὸ Γ ,
ἢ δὲ δοθεῖσα γωνία εὐθύγραμμος ἢ Δ .

δεῖ δὴ
παρὰ τὴν δοθεῖσαν εὐθεῖαν τὴν AB
τῷ δοθέντι τριγώνῳ τῷ Γ ἴσον
παράλληλόγραμμον παραβαλεῖν
ἐν ἴσῃ τῇ Δ γωνίᾳ.

Συνεστάτω
τῷ Γ τριγώνῳ ἴσον
παράλληλόγραμμον τὸ BEZH
ἐν γωνίᾳ τῇ ὑπὸ EBH,
ἣ ἔστιν ἴση τῇ Δ .
καὶ κείσθω
ὥστε ἐπ' εὐθείας εἶναι τὴν BE
τῇ AB,
καὶ διήχθω
ἢ ZH ἐπὶ τὸ Θ ,
καὶ διὰ τοῦ A
ὁποτέρᾳ τῶν BH, EZ
παράλληλος ἦχθω ἢ A Θ ,
καὶ ἐπεζεύχθω ἢ Θ B.

καὶ ἐπεὶ εἰς παραλλήλους τὰς A Θ , EZ
εὐθεῖα ἐνέπεσεν ἢ Θ Z,
αἱ ἄρα ὑπὸ A Θ Z, Θ ZE γωνίαι
δυσὶν ὀρθαῖς εἰσὶν ἴσαι.
αἱ ἄρα ὑπὸ B Θ H, HZE
δύο ὀρθῶν ἐλάσσονές εἰσιν·
αἱ δὲ ἀπὸ ἐλασσόνων ἢ δύο ὀρθῶν εἰς
ἄπειρον ἐκβαλλόμεναι
συμπίπτουσιν·
αἱ Θ B, ZE ἄρα ἐκβαλλόμεναι
συμπεσοῦνται.

ἐκβεβλήσθωσαν
καὶ συμπίπτέτωσαν κατὰ τὸ K,
καὶ διὰ τοῦ K σημείου
ὁποτέρᾳ τῶν EA, Z Θ παράλληλος
ἦχθω ἢ K Λ ,
καὶ ἐκβεβλήσθωσαν αἱ Θ A, HB
ἐπὶ τὰ Λ , M σημεία.

παράλληλόγραμμον ἄρα ἔστι τὸ Θ Λ KZ,
διάμετρος δὲ αὐτοῦ ἢ Θ K,
περὶ δὲ τὴν Θ K
παράλληλόγραμμα μὲν τὰ AH, ME,
τὰ δὲ λεγόμενα παραπληρώματα
τὰ Λ B, BZ·
ἴσον ἄρα ἔστι τὸ Λ B τῷ BZ.
ἀλλὰ τὸ BZ τῷ Γ τριγώνῳ ἔστιν ἴσον·
καὶ τὸ Λ B ἄρα τῷ Γ ἔστιν ἴσον.
καὶ ἐπεὶ ἴση ἔστιν
ἢ ὑπὸ HBE γωνία τῇ ὑπὸ ABM,
ἀλλὰ ἢ ὑπὸ HBE τῇ Δ ἔστιν ἴση,
καὶ ἢ ὑπὸ ABM ἄρα τῇ Δ γωνίᾳ

ve verilen düzkenar açı Δ olsun.

Şimdi gereklidir
verilen AB doğrusu boyunca
 Γ üçgenine eşit
bir paralelkenarı
 Δ açısında yerleştirmek.

Varsayılsın inşa edildiği
 Γ üçgenine eşit,
bir BEZH paralelkenarının
EBH açısında,
eşit olan Δ açısına;
ve öyle yerleştirilmiş olsun ki
bir doğruya kalsın BE,
AB ile,
ve çizilmiş olsun
ZH doğrusundan Θ noktasına,
ve A noktasından,
paralel olan BH ve EZ doğrularından
birine,
çizilmiş olsun
A Θ ,
ve birleştirilmiş olsun
 Θ B.

Ve A Θ ile EZ paralellerinin üzerine
düştüğünden Θ Z doğrusu,
A Θ Z ve Θ ZE açıları
eşittir iki dik açıya.
Dolayısıyla B Θ H ve HZE
küçüktür iki dik açıdan.
Ve küçük olanlardan
iki dik açıdan,
uzatıldıklarında sonsuza,
birbirlerine düşerler doğrular.
Dolayısıyla Θ B ve ZE, uzatılırsa,
birbirlerine düşerler.

Varsayılsın uzatıldıkları,
ve K noktasında kesiştikleri,
ve K noktasından,
paralel olan EA veya Z Θ doğrusuna,
çizilmiş olsun K Λ ,
ve uzatılmış olsunlar Θ A ve HB doğru-
ları
 Λ ve M noktalarından.

Bir paralelkenardır dolayısıyla Θ Λ KZ,
ve onun köşegeni Θ K,
ve Θ K etrafındadır
AH ve ME paralelkenarları,
ve bunların tümleyenleris,
 Λ B ile BZ;
eşittirler dolayısıyla Λ B ile BZ
tümleyenlerine.
Ama BZ, Γ üçgenine eşittir.
Dolayısıyla Λ B da Γ üçgenine eşittir.
Ve eşit olduğundan
HBE, ABM açısına,
fakat HBE, Δ açısına eşit,

is equal.

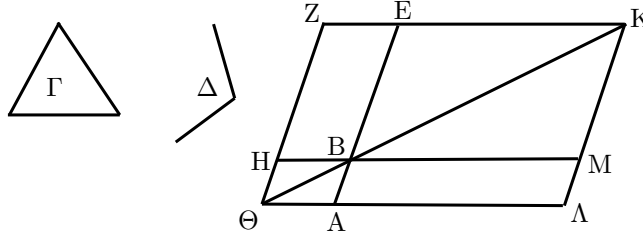
Therefore, along the given STRAIGHT, AB, equal to the given triangle, Γ , a parallelogram has been applied, ΛB , in the angle ABM, which is equal to Δ ; —just what it was necessary to do.

ἔστιν ἴση.

Παρά τὴν δοθεῖσαν ἄρα εὐθεΐαν τὴν AB τῷ δοθέντι τριγώνῳ τῷ Γ ἴσον παραλληλόγραμμον παραβέβληται τὸ ΛB ἐν γωνίᾳ τῇ ὑπὸ ABM, ἣ ἔστιν ἴση τῇ Δ . ὅπερ ἔδει ποιῆσαι.

dolayısıyla ABM de Δ açısına eşittir.

Dolayısıyla, verilen bir, AB doğrusu boyunca, verilen bir Γ üçgenine eşit, bir, ΛB paralelkenarı yerleştirilmiş oldu, ABM açısında, eşit olan Δ açısına; —yapılması gereken tam buydu.



1.45

To the given rectilinear [figure] equal a parallelogram to construct in the given rectilinear angle.

Let be the given rectilinear [figure] $AB\Gamma\Delta$, and the given rectilinear angle, E.

It is necessary then to the rectilinear $AB\Gamma\Delta$ equal a parallelogram to construct in the given angle E.

Suppose has been joined ΔB , and suppose has been constructed, equal to the triangle $AB\Delta$, a parallelogram, $Z\Theta$, in the angle ΘKZ , which is equal to E; and suppose there has been applied along the STRAIGHT $H\Theta$, equal to triangle $\Delta B\Gamma$, a parallelogram, HM , in the angle $H\Theta M$, which is equal to E.

And since angle E to either of ΘKZ and $H\Theta M$ is equal, therefore also ΘKZ to $H\Theta M$ is equal. Let $K\Theta H$ be added in common; therefore $ZK\Theta$ and $K\Theta H$ to $K\Theta H$ and $H\Theta M$ are equal. But $ZK\Theta$ and $K\Theta H$ are equal to two RIGHTS; therefore also $K\Theta H$ and $H\Theta M$ are equal to two RIGHTS.

Τῷ δοθέντι εὐθύγραμμῳ ἴσον παραλληλόγραμμον συστήσασθαι ἐν τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ.

Ἐστω τὸ μὲν δοθὲν εὐθύγραμμον τὸ $AB\Gamma\Delta$, ἡ δὲ δοθεῖσα γωνία εὐθύγραμμος ἡ E.

δεῖ δὴ τῷ $AB\Gamma\Delta$ εὐθύγραμμῳ ἴσον παραλληλόγραμμον συστήσασθαι ἐν τῇ δοθείσῃ γωνίᾳ τῇ E.

Ἐπεζεύχθω ἡ ΔB , καὶ συνεστάτω τῷ $AB\Delta$ τριγώνῳ ἴσον παραλληλόγραμμον τὸ $Z\Theta$ ἐν τῇ ὑπὸ ΘKZ γωνίᾳ, ἣ ἔστιν ἴση τῇ E· καὶ παραβέβλησθω παρά τὴν $H\Theta$ εὐθεΐαν τῷ $\Delta B\Gamma$ τριγώνῳ ἴσον παραλληλόγραμμον τὸ HM ἐν τῇ ὑπὸ $H\Theta M$ γωνίᾳ, ἣ ἔστιν ἴση τῇ E.

καὶ ἐπεὶ ἡ E γωνία ἑκατέρω τῶν ὑπὸ ΘKZ , $H\Theta M$ ἔστιν ἴση, καὶ ἡ ὑπὸ ΘKZ ἄρα τῇ ὑπὸ $H\Theta M$ ἔστιν ἴση. κοινὴ προσχείσθω ἡ ὑπὸ $K\Theta H$. αἱ ἄρα ὑπὸ $ZK\Theta$, $K\Theta H$ ταῖς ὑπὸ $K\Theta H$, $H\Theta M$ ἴσαι εἰσίν. ἀλλ' αἱ ὑπὸ $ZK\Theta$, $K\Theta H$ δυσὶν ὀρθαῖς ἴσαι εἰσίν· καὶ αἱ ὑπὸ $K\Theta H$, $H\Theta M$ ἄρα δύο ὀρθαῖς ἴσαι εἰσίν.

Verilen bir düzkenar [figüre] eşit bir paralelkenar inşa etmek, verilen düzkenar açıda.

Verilmiş olsun $AB\Gamma\Delta$ düzkenar [figürü], ve düzkenar E açısı.

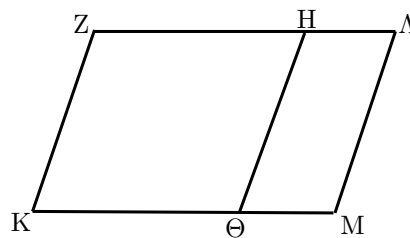
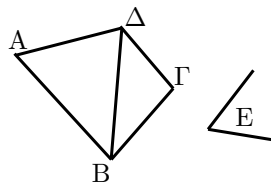
Gereklidir şimdi $AB\Gamma\Delta$ düzkenarına eşit bir paralelkenar inşa etmek, verilen E açısında.

Birleştirilmiş olduğu ΔB doğrusunun, ve inşa edilmiş olsun, $AB\Delta$ üçgenine eşit, bir $Z\Theta$ paralelkenarı, ΘKZ açısında, eşit olan E açısına; ve yerleştirilmiş olsun $H\Theta$ doğrusu boyunca, $\Delta B\Gamma$ üçgenine eşit, bir HM paralelkenarı, $H\Theta M$ açısında, eşit olan E açısına.

Ve E açısı ΘKZ ve $H\Theta M$ açılarının her birine eşit olduğundan, ΘKZ da $H\Theta M$ açısına eşittir. Eklenmiş olsun $K\Theta H$ ortak olarak; dolayısıyla $ZK\Theta$ ve $K\Theta H$, $K\Theta H$ ve $H\Theta M$ açılara eşittirler. Fakat $ZK\Theta$ ve $K\Theta H$ eşittirler iki dik açıya; dolayısıyla $K\Theta H$ ve $H\Theta M$ açılarda eşittirler iki dik açıya.

Then to some STRAIGHT, $H\Theta$,
and at the same point, Θ ,
two STRAIGHTS, $K\Theta$ and ΘM ,
not lying in the same parts,
the adjacent angles
make equal to two RIGHTS.
In a STRAIGHT then are $K\Theta$ and ΘM ;
and since on the parallels KM and ZH
fell the STRAIGHT ΘH ,
the alternate angles $M\Theta H$ and $\Theta H Z$
are equal to one another.
Let $\Theta H A$ be added in common;
therefore $M\Theta H$ and $\Theta H A$
to $\Theta H Z$ and $\Theta H A$
are equal.
But $M\Theta H$ and $\Theta H A$
are equal to two RIGHTS;
therefore also $\Theta H Z$ and $\Theta H A$
are equal to two RIGHTS;
therefore on a STRAIGHT are ZH and
 $H A$.
And since ZK to ΘH
is equal and parallel,
but also ΘH to $M A$,
therefore also KZ to $M A$
is equal and parallel;
and join them
 KM and $Z A$, which are STRAIGHTS;
therefore also KM and $Z A$
are equal and parallel;
a parallelogram therefore is $KZ A M$.
And since equal is
triangle $A B \Delta$
to the parallelogram $Z \Theta$,
and $\Delta B \Gamma$ to $H M$,
therefore, as a whole,
the rectilinear $A B \Gamma \Delta$
to parallelogram $KZ A M$ as a whole
is equal.

Therefore, to the given rectilinear [fig-
ure], $A B \Gamma \Delta$, equal,
a parallelogram has been constructed,
 $KZ A M$,
in the angle $Z K M$,
which is equal to the given E ;
—just what it was necessary to do.



πρὸς δὴ τινι εὐθεῖα τῇ $H\Theta$
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Θ
δύο εὐθεῖαι αἱ $K\Theta$, ΘM
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δύο ὀρθαῖς ἴσας ποιοῦσιν·
ἐπ' εὐθείας ἄρα ἐστὶν ἡ $K\Theta$ τῇ ΘM ·
καὶ ἐπεὶ εἰς παραλλήλους τὰς KM , ZH
εὐθεῖα ἐνέπεσεν ἡ ΘH ,
αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ $M\Theta H$, $\Theta H Z$
ἴσαι
ἀλλήλαις εἰσίν.
κοινὴ προσκείσθω ἡ ὑπὸ $\Theta H A$ ·
αἱ ἄρα ὑπὸ $M\Theta H$, $\Theta H A$ ταῖς ὑπὸ $\Theta H Z$,
 $\Theta H A$
ἴσαι εἰσίν.
ἀλλ' αἱ ὑπὸ $M\Theta H$, $\Theta H A$
δύο ὀρθαῖς ἴσαι εἰσίν·
καὶ αἱ ὑπὸ $\Theta H Z$, $\Theta H A$ ἄρα
δύο ὀρθαῖς ἴσαι εἰσίν·
ἐπ' εὐθείας ἄρα ἐστὶν ἡ ZH τῇ $H A$.
καὶ ἐπεὶ ἡ ZK τῇ ΘH
ἴση τε καὶ παράλληλος ἐστίν,
ἀλλὰ καὶ ἡ ΘH τῇ $M A$,
καὶ ἡ KZ ἄρα τῇ $M A$
ἴση τε καὶ παράλληλος ἐστίν·
καὶ ἐπιζευγνύουσιν αὐτὰς εὐθεῖαι αἱ
 KM , $Z A$ ·
καὶ αἱ KM , $Z A$ ἄρα
ἴσαι τε καὶ παράλληλοι εἰσίν·
παραλληλόγραμμον ἄρα ἐστὶ τὸ
 $KZ A M$.
καὶ ἐπεὶ ἴσον ἐστὶ
τὸ μὲν $A B \Delta$ τρίγωνον τῷ $Z \Theta$ παρα-
λληλογράμμῳ,
τὸ δὲ $\Delta B \Gamma$ τῷ $H M$,
ὅλον ἄρα τὸ $A B \Gamma \Delta$ εὐθύγραμμον
ὅλῳ τῷ $KZ A M$ παραλληλογράμμῳ
ἐστὶν ἴσον.

Τῷ ἄρα δοθέντι εὐθυγράμμῳ τῷ $A B \Gamma \Delta$
ἴσον
παραλληλόγραμμον συνέσταται
τὸ $KZ A M$
ἐν γωνίᾳ τῇ ὑπὸ $Z K M$,
ἣ ἐστὶν ἴση τῇ δοθείσῃ τῇ E ·
ὅπερ ἔδει ποιῆσαι.

Şimdi bir $H\Theta$ doğrusuna,
ve aynı Θ noktasında,
iki $K\Theta$ ve ΘM doğruları,
aynı tarafta kalmayan,
komşu açıları
iki dik açiya eşit yapar.
O zaman bir doğrudadır $K\Theta$ ve ΘM ;
ve KM ve ZH paralelleri üzerine
düştüğünden ΘH doğrusu,
ters $M\Theta H$ ve $\Theta H Z$ açıları
eşittir birbirine.
eklenmiş olsun $\Theta H A$ ortak olarak;
dolayısıyla $M\Theta H$ ve $\Theta H A$,
 $\Theta H Z$ ve $\Theta H A$ açılarına
eşittirler.
Fakat $M\Theta H$ ve $\Theta H A$
eşittirler iki dik açiya;
dolayısıyla $\Theta H Z$ ve $\Theta H A$ da
eşittirler iki dik açiya;
dolayısıyla bir doğru üzerindedir ZH
ve $H A$.
Ve olduğundan ZK , ΘH doğrusuna
eşit ve paralel,
ve de ΘH , $M A$ doğrusuna,
dolayısıyla KZ da $M A$ doğrusuna
eşit ve paraleldir;
ve birleştirir onları KM ile $Z A$, ki bun-
larda doğrulardır;
dolayısıyla KM ve $Z A$ da
eşit ve paraleldirler;
dolayısıyla $KZ A M$ bir paralelkenardır.
Ve eşit olduğundan
 $A B \Delta$ üçgeni
 $Z \Theta$ paralelkenarına,
ve $\Delta B \Gamma$, $H M$ paralelkenarına,
dolayısıyla, bir bütün olarak,
 $A B \Gamma \Delta$ düzkenarı
bir bütün olarak $KZ A M$ paralelke-
narına
eşittir.

Dolayısıyla, verilen düzkenar $A B \Gamma \Delta$
figürüne eşit,
bir $KZ A M$ paralelkenarı inşa edilmiş
oldu,
 $Z K M$ açısında,
eşit olan verilmiş E açısına;
—yapılması gereken tam buydu.

1.46

On the given STRAIGHT
to set up a square.

Let be
the given STRAIGHT AB.

It is required then
on the STRAIGHT AB
to set up a square.

Suppose there has been drawn
to the STRAIGHT AB,
at the point A of it,
at a RIGHT,
ΑΓ,
and suppose there has been laid down,
equal to AB,
ΑΔ;
and through the point Δ,
parallel to AB,
suppose there has been drawn ΔΕ;
and through the point Β,
parallel to ΑΔ,
suppose there has been drawn ΒΕ.

A parallelogram therefore is ΑΔΕΒ;
equal therefore is ΑΒ to ΔΕ,
and ΑΔ to ΒΕ.
But ΑΒ to ΑΔ is equal.
Therefore the four
ΒΑ, ΑΔ, ΔΕ, and ΕΒ
are equal to one another;
equilateral therefore
is the parallelogram ΑΔΕΒ.

I say then that
it is also right-angled.

For, since on the parallels ΑΒ and ΔΕ
fell the STRAIGHT ΑΔ,
therefore the angles ΒΑΔ and ΑΔΕ
are equal to two RIGHTS.
And ΒΑΔ is right;
right therefore is ΑΔΕ.
And of parallelogram areas
the opposite sides and angles
are equal to one another.
Right therefore is either
of the opposite angles ΑΒΕ and ΒΕΔ;
right-angled therefore is ΑΔΕΒ.
And it was shown also equilateral.

A square therefore it is;
and it is on the STRAIGHT ΑΒ
set up;
—just what it was necessary to do.

Ἀπὸ τῆς δοθείσης εὐθείας
τετράγωνον ἀναγράψαι.

Ἐστω
ἡ δοθεῖσα εὐθεῖα ἡ ΑΒ·

δεῖ δὴ
ἀπὸ τῆς ΑΒ εὐθείας
τετράγωνον ἀναγράψαι.

Ἦχθω
τῆ ΑΒ εὐθείᾳ
ἀπὸ τοῦ πρὸς αὐτῇ σημείου τοῦ Α
πρὸς ὀρθὰς
ἡ ΑΓ,
καὶ κείσθω
τῆ ΑΒ ἴση
ἡ ΑΔ·
καὶ διὰ μὲν τοῦ Δ σημείου
τῆ ΑΒ παράλληλος
ἦχθω ἡ ΔΕ,
διὰ δὲ τοῦ Β σημείου
τῆ ΑΔ παράλληλος
ἦχθω ἡ ΒΕ.

παρὰλληλόγραμμον ἄρα ἐστὶ τὸ ΑΔΕΒ·
ἴση ἄρα ἐστὶν ἡ μὲν ΑΒ τῆ ΔΕ,
ἡ δὲ ΑΔ τῆ ΒΕ.
ἀλλὰ ἡ ΑΒ τῆ ΑΔ ἐστὶν ἴση·
αἱ τέσσαρες ἄρα
αἱ ΒΑ, ΑΔ, ΔΕ, ΕΒ
ἴσαι ἀλλήλαις εἰσὶν·
ἰσόπλευρον ἄρα
ἐστὶ τὸ ΑΔΕΒ παρὰλληλόγραμμον.

λέγω δὴ, ὅτι
καὶ ὀρθογώνιον.

ἐπεὶ γὰρ εἰς παρὰλληλους τὰς ΑΒ, ΔΕ
εὐθεῖα ἐνέπεσεν ἡ ΑΔ,
αἱ ἄρα ὑπὸ ΒΑΔ, ΑΔΕ γωνίαι
δύο ὀρθαῖς ἴσαι εἰσὶν.
ὀρθὴ δὲ ἡ ὑπὸ ΒΑΔ·
ὀρθὴ ἄρα καὶ ἡ ὑπὸ ΑΔΕ.
τῶν δὲ παρὰλληλογράμμων χωρίων
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσὶν·
ὀρθὴ ἄρα καὶ ἑκατέρω
τῶν ἀπεναντίον τῶν ὑπὸ ΑΒΕ, ΒΕΔ
γωνιῶν·
ὀρθογώνιον ἄρα ἐστὶ τὸ ΑΔΕΒ.
ἐδείχθη δὲ καὶ ἰσόπλευρον.

Τετράγωνον ἄρα ἐστὶν·
καὶ ἐστὶν ἀπὸ τῆς ΑΒ εὐθείας
ἀναγεγραμμένον·
ὅπερ ἔδει ποιῆσαι.

Verilen bir doğrudaki
bir kare kurmak.

Verilmiş olsun
ΑΒ doğrusu.

Şimdi gereklidir
ΑΒ doğrusunda
bir kare kurmak.

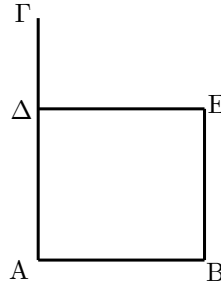
Çizilmiş olsun
ΑΒ doğrusunda,
onun Α noktasında,
dik açıda,
ΑΓ,
ve yerleştirilmiş olsun,
ΑΒ doğrusuna eşit,
ΑΔ;
ve Δ noktasından,
ΑΒ doğrusuna paralel,
çizilmiş olsun ΔΕ;
ve Β noktasından,
ΑΔ doğrusuna paralel,
ΒΕ çizilmiş olsun.

Bir paralelkenardır dolayısıyla
ΑΔΕΒ;
eşittir dolayısıyla ΑΒ, ΔΕ doğrusuna,
ve ΑΔ, ΒΕ doğrusuna.
Ama ΑΒ, ΑΔ doğrusuna eşittir.
Dolayısıyla şu dördü
ΒΑ, ΑΔ, ΔΕ ve ΕΒ
birbirlerine eşittirler;
eşkenardır dolayısıyla
ΑΔΕΒ paralelkenarı.

Şimdi iddia ediyorum ki
aynı zamanda dik açıdır.

Çünkü, ΑΒ ve ΔΕ paralellerinin üzeri-
ne
düştüğünden ΑΔ doğrusu,
eşittir dolayısıyla ΒΑΔ ve ΑΔΕ
iki dik açıya.
Ve ΒΑΔ diktir;
diktir dolayısıyla ΑΔΕ.
Ve paralelkenar alanların
karşıt kenar ve açıları
eşittir birbirlerine.
Diktir dolayısıyla her bir
karşıt açı ΑΒΕ ve ΒΕΔ;
dik açıdır dolayısıyla ΑΔΕΒ.
Ve gösterilmişti ki eşkenardır da.

Bir karedir dolayısıyla o;
ve o ΑΒ doğrusu üzerine
kurulmuştur;
—yapılması gereken tam buydu.



1.47

In right-angled triangles,
the square on the side that subtends
the right angle
is equal
to the squares on the sides that con-
tain the right angle.

Let be
a right-angled triangle, ABΓ,
having the angle BAG right.

I say that
the square on ΓB
is equal
to the squares on BA and AΓ.

For, suppose there has been set up
on BΓ
a square, BΔΕΓ,
and on BA and AΓ,
HB and ΘΓ,
and through A,
parallel to either of BΔ and ΓΕ,
suppose AA has been drawn;
and suppose have been joined
AΔ and ZΓ.

And since right is
either of the angles BAG and BAH,
on some STRAIGHT, BA,
to the point A on it,
two STRAIGHTS, AΓ and AH,
not lying in the same parts,
the adjacent angles
make equal to two RIGHTS;
on a STRAIGHT therefore is ΓA with
AH.

Then for the same [reason]
also BA with AΘ is on a STRAIGHT.
And since equal is
angle ΔBΓ to angle ZBA;
for either is RIGHT;
let ABΓ be added in common;
therefore ΔBA as a whole
to ZBΓ as a whole
is equal.
And since equal is

Ἐν τοῖς ὀρθογωνίοις τριγώνοις
τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτε-
νοῦσης πλευρᾶς τετραγώνον
ἴσον ἐστὶ
τοῖς ἀπὸ τῶν τὴν ὀρθὴν γωνίαν περιε-
χουσῶν πλευρῶν τετραγώνοις.

Ἐστω
τρίγωνον ὀρθογώνιον τὸ ABΓ
ὀρθὴν ἔχον τὴν ὑπὸ BAG γωνίαν·

λέγω, ὅτι
τὸ ἀπὸ τῆς BΓ τετραγώνον
ἴσον ἐστὶ
τοῖς ἀπὸ τῶν BA, AΓ τετραγώνοις.

Ἀναγεγράφθω γὰρ
ἀπὸ μὲν τῆς BΓ
τετραγώνον τὸ BΔΕΓ,
ἀπὸ δὲ τῶν BA, AΓ
τὰ HB, ΘΓ,
καὶ διὰ τοῦ A
ὀποτέρᾳ τῶν BΔ, ΓΕ παράλληλος
ἦχθω ἡ AA.¹
καὶ ἐπεζεύχθωσαν
αἱ AΔ, ZΓ.

καὶ ἐπεὶ ὀρθὴ ἐστὶν
ἑκάτερα τῶν ὑπὸ BAG, BAH γωνιῶν,
πρὸς δὴ τινὶ εὐθείᾳ τῆς BA
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A
δύο εὐθεῖαι αἱ AΓ, AH
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δυσὶν ὀρθαῖς ἴσας ποιῶσιν·
ἐπ' εὐθείας ἄρα ἐστὶν ἡ ΓA τῆς AH.
διὰ τὰ αὐτὰ δὲ
καὶ ἡ BA τῆς AΘ ἐστὶν ἐπ' εὐθείας.
καὶ ἐπεὶ ἴση ἐστὶν
ἡ ὑπὸ ΔBΓ γωνία τῆς ὑπὸ ZBA·
ὀρθὴ γὰρ ἑκάτερα·
κοινὴ προσκείμεθα ἡ ὑπὸ ABΓ·
ὄλη ἄρα ἡ ὑπὸ ΔBA
ὄλη τῆς ὑπὸ ZBΓ
ἐστὶν ἴση.
καὶ ἐπεὶ ἴση ἐστὶν
ἡ μὲν ΔB τῆς BΓ,

Dik açılı üçgenlerde,
dik açının gördüğü kenar üzerindeki
kare
eşittir
dik açığı içeren kenarların üzerindeki
karelere.

Verilmiş olsun
dik açılı bir ABΓ üçgeni
BAG açısı dik olan.

İddia ediyorum ki
ΓB üzerindeki kare
eşittir
BA ve AΓ üzerlerindeki karelere.

Çünkü, kurulmuş olsun
BΓ üzerinde
bir BΔΕΓ karesi,
ve BA ile AΓ üzerlerinde,
HB ve ΘΓ,
ve A noktasından,
BΔ ve ΓΕ doğrularına paralel olan,
AA çizilmiş olsun;
ve birleştirilmiş olsun
AΔ ve ZΓ.

Ve dik olduğundan
BAG ve BAH açılarının her biri,
bir BA doğrusunda,
üzerindeki A noktasına,
AΓ ve AH doğruları,
aynı tarafta kalmayan,
bitişik açılar
oluştururlar eşit iki dik açığa;
bir doğrudadır dolayısıyla ΓA ile AH.
Sonra aynı nedenle
BA ile AΘ da bir doğrudadır.
Ve eşit olduğundan
ΔBΓ, ZBA açısına;
her ikisinde dikdir;
eklenmiş olsun ABΓ her ikisine de;
dolayısıyla ΔBA açısının tamamı
ZBΓ açısının tamamına
eşittir.
Ve eşit olduğundan
ΔB, BΓ doğrusuna,

¹Heiberg's text [1, p. 110] has Δ for Λ at this place and else-
where (though not in the diagram). Probably this is a compositor's

mistake, owing to the similarity in appearance of the two letters,
especially in the font used.

ΔB to $B\Gamma$,
and ZB to BA ,
the two ΔB and BA
to the two ZB and $B\Gamma$ ²
are equal,
either to either;
and angle ΔBA
to angle $ZB\Gamma$
is equal;
therefore the base AA
to the base $Z\Gamma$
[is] equal,
and the triangle $AB\Delta$
to the triangle $ZB\Gamma$
is equal;
and of the triangle $AB\Delta$
the parallelogram BA is double;
for they have the same base, BA ,
and are in the same parallels,
 $B\Delta$ and AA ;
and of the triangle $ZB\Gamma$
the square HB is double;
for again they have the same base,
 ZB ,
and are in the same parallels,
 ZB and $H\Gamma$.
[And of equals,
the doubles are equal to one another.]
Equal therefore is
also the parallelogram BA
to the square HB .
Similarly then,
there being joined AE and BK ,
it will be shown that
also the parallelogram ΓA
[is] equal to the square $\Theta\Gamma$.
Therefore the square $\Delta BE\Gamma$ as a
whole
to the two squares HB and $\Theta\Gamma$
is equal.
Also is
the square $B\Delta E\Gamma$ set up on $B\Gamma$,
and HB and $\Theta\Gamma$ on BA and $A\Gamma$.
Therefore the square on the side $B\Gamma$
is equal
to the squares on the sides BA and
 $A\Gamma$.

Therefore in right-angled triangles
the square on the side subtending the
right angle
is equal
to the squares on the sides subtending
the right [angle];
—just what it was necessary to show.

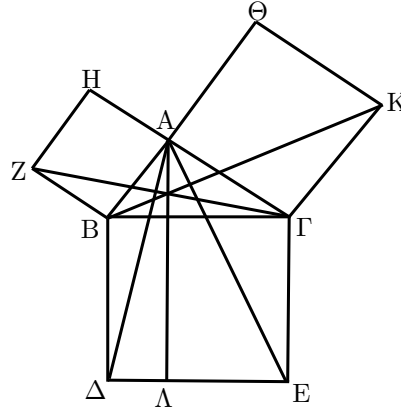
ἡ δὲ ZB τῆ BA ,
δύο δὴ αἱ ΔB , BA
δύο ταῖς ZB , $B\Gamma$
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρᾳ·
καὶ γωνία ἡ ὑπὸ ΔBA
γωνία τῆ ὑπὸ $ZB\Gamma$
ἴση·
βάσις ἄρα ἡ $A\Delta$
βάσει τῆ $Z\Gamma$
[ἔστιν] ἴση,
καὶ τὸ $AB\Delta$ τρίγωνον
τῷ $ZB\Gamma$ τριγώνῳ
ἔστιν ἴσον·
καὶ [ἔστι] τοῦ μὲν $AB\Delta$ τριγώνου
διπλάσιον τὸ BA παραλληλόγραμμον·
βάσιν τε γὰρ τὴν αὐτὴν ἔχουσι τὴν $B\Delta$
καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις
ταῖς $B\Delta$, AA ·
τοῦ δὲ $ZB\Gamma$ τριγώνου
διπλάσιον τὸ HB τετράγωνον·
βάσιν τε γὰρ πάλιν τὴν αὐτὴν ἔχουσι
τὴν ZB
καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις
ταῖς ZB , $H\Gamma$.
[τὰ δὲ τῶν ἴσων
διπλάσια ἴσα ἀλλήλοις ἔστιν.]
ἴσον ἄρα ἔστι
καὶ τὸ BA παραλληλόγραμμον
τῷ HB τετραγώνῳ.
ὁμοίως δὴ
ἐπιζευγνυμένων τῶν AE , BK
δειχθήσεται
καὶ τὸ ΓA παραλληλόγραμμον
ἴσον τῷ $\Theta\Gamma$ τετραγώνῳ·
ὄλον ἄρα τὸ $B\Delta E\Gamma$ τετράγωνον
δυοῖς τοῖς HB , $\Theta\Gamma$ τετραγώνοις
ἴσον ἔστιν.
καὶ ἔστι
τὸ μὲν $B\Delta E\Gamma$ τετράγωνον ἀπὸ τῆς $B\Gamma$
ἀναγραφέν,
τὰ δὲ HB , $\Theta\Gamma$ ἀπὸ τῶν BA , $A\Gamma$.
τὸ ἄρα ἀπὸ τῆς $B\Gamma$ πλευρᾶς τετράγω-
νον
ἴσον ἔστι
τοῖς ἀπὸ τῶν BA , $A\Gamma$ πλευρῶν τε-
τραγώνοις.

Ἐν ἄρα τοῖς ὀρθογωνίοις τριγώνοις
τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτεί-
νούσης πλευρᾶς τετράγωνον
ἴσον ἔστι
τοῖς ἀπὸ τῶν τὴν ὀρθὴν [γωνίαν] περιε-
χουσῶν πλευρῶν τετραγώνοις·
ὄπερ ἔδει δεῖξαι.

ve ZB , BA doğrusuna
 ΔB ve BA ikilisi
 ZB ve $B\Gamma$ ikilisine³
eşittirler,
her biri birine;
ve ΔBA açısı
 $ZB\Gamma$ açısına
eşittir;
dolayısıyla AA tabanı
 $Z\Gamma$ tabanına
eşittir,
ve $AB\Delta$ üçgeni
 $ZB\Gamma$ üçgenine
eşittir;
ve $AB\Delta$ üçgeninin
 BA paralelkenarı iki katıdır;
aynı BA tabanları olduğu,
ve aynı
 $B\Delta$ ve AA paralelerinde oldukları için;
ve $ZB\Gamma$ üçgeninin
 HB karesi iki katıdır;
yine aynı
 ZB tabanları olduğu
ve aynı
 ZB ve $H\Gamma$ paralelerinde oldukları için.
[Ve eşitlerin,
iki katları birbirlerine eşittirler.]
Eşittir dolayısıyla
 BA paralelkenarı da
 HB karesine.
Şimdi benzer şekilde,
birleştirildiğinde AE ve BK ,
gösterilecek ki
 ΓA paralelkenarı da
eşittir $\Theta\Gamma$ karesine.
Dolayısıyla $\Delta BE\Gamma$ bir bütün olarak
 HB ve $\Theta\Gamma$ iki karesine
eşittir.
Ayrıca
 $B\Delta E\Gamma$ karesi $B\Gamma$ üzerine kurulmuştur,
ve HB ve $\Theta\Gamma$, BA ve $A\Gamma$ üzerine.
Dolayısıyla $B\Gamma$ kenarındaki kare
eşittir
 BA ve $A\Gamma$ kenarlarındaki karelere.

Dolayısıyla dik açılı üçgenlerde,
dik açının gördüğü kenar üzerindeki
kare
eşittir
dik açığı içeren kenarların üzerindeki-
lere;
—gösterilmesi gereken tam buydu.

²Fitzpatrick considers this ordering of the two straight lines to be 'obviously a mistake'. But if it is a mistake, how could it have been made?



1.48

If of a triangle
the square on one of the sides
be equal
to the squares on the remaining sides
of the triangle,
the angle contained
by the two remaining sides of the tri-
angle
is right.

For, of the triangle ABΓ
the square on the one side BΓ
—suppose it is equal
to the squares on the sides BA and
ΑΓ.

I say that
right is the angle BΑΓ.

For, suppose has been drawn
from the point A
to the STRAIGHT ΑΓ
at RIGHTS
ΑΔ,
and let be laid down
equal to BA
ΑΔ,
and suppose ΔΓ has been joined.

Since equal is ΔΑ to AB,
equal is
also the square on ΔΑ
to the square on AB.
Let be added in common
the square on ΑΓ;
therefore the squares on ΔΑ and ΑΓ
are equal
to the squares on BA and ΑΓ.
But those on ΔΑ and ΑΓ
are equal
to that on ΔΓ;
for right is the angle ΔΑΓ;
and those on BA and ΑΓ
are equal

Ἐὰν τριγώνου
τὸ ἀπὸ μιᾶς τῶν πλευρῶν τετράγωνον
ἴσον ᾗ
τοῖς ἀπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
πλευρῶν τετραγώνοις,
ἡ περιεχομένη γωνία
ὑπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
πλευρῶν
ὀρθή ἐστίν.

Τριγώνου γὰρ τοῦ ABΓ
τὸ ἀπὸ μιᾶς τῆς BΓ πλευρᾶς τετράγω-
νον
ἴσον ἔστω
τοῖς ἀπὸ τῶν BA, ΑΓ πλευρῶν τε-
τραγώνοις·

λέγω, ὅτι
ὀρθή ἐστίν ἡ ὑπὸ BΑΓ γωνία.

Ἦχθω γὰρ
ἀπὸ τοῦ A σημείου
τῆς ΑΓ εὐθείᾳ
πρὸς ὀρθὰς
ἡ ΑΔ
καὶ κείσθω
τῆς BA ἴση
ἡ ΑΔ,
καὶ ἐπεζεύχθω ἡ ΔΓ.

ἐπεὶ ἴση ἐστίν ἡ ΔΑ τῆς AB,
ἴσον ἐστὶ
καὶ τὸ ἀπὸ τῆς ΔΑ τετράγωνον
τῷ ἀπὸ τῆς AB τετραγώνῳ.
κοινὸν προσκείσθω
τὸ ἀπὸ τῆς ΑΓ τετράγωνον·
τὰ ἄρα ἀπὸ τῶν ΔΑ, ΑΓ τετράγωνα
ἴσα ἐστὶ
τοῖς ἀπὸ τῶν BA, ΑΓ τετραγώνοις.
ἀλλὰ τοῖς μὲν ἀπὸ τῶν ΔΑ, ΑΓ
ἴσον ἐστὶ
τὸ ἀπὸ τῆς ΔΓ·
ὀρθή γὰρ ἐστίν ἡ ὑπὸ ΔΑΓ γωνία·
τοῖς δὲ ἀπὸ τῶν BA, ΑΓ
ἴσον ἐστὶ

Eğer bir üçgende
bir kenarın üzerindeki kare
eşitse
üçgenin geriye kalan kenarlarındaki
karelere,
üçgenin geriye kalan kenarlarınca içe-
rilen
açı
dikdir.

Çünkü, ABΓ üçgeninin
bir BΓ kenarındaki karesi
—varsayılınsın eşit
BA ve ΑΓ kenarlarındaki karelere.

İddia ediyorum ki
BΑΓ açısı dikdir.

Çünkü, çizilmiş olsun
A noktasından
ΑΓ doğrusuna
dik açılarda
ΑΔ,
ve yerleştirilmiş olsun
BA doğrusuna eşit
ΑΔ,
ve ΔΓ birleştirilmiş olsun.

Eşit olduğundan ΔΑ, AB kenarına,
eşittir
ΔΑ üzerindeki kare de
AB üzerindeki kareye.
Eklenmiş olsun ortak
ΑΓ üzerindeki kare;
dolayısıyla ΔΑ ve ΑΓ üzerlerindeki
kareler
eşittir
BA ve ΑΓ üzerlerindeki karelere.
Ama ΔΑ ve ΑΓ kenarları üzer-
lerindeki
eşittir
ΔΓ üzerlerindeki;
ΔΑΓ açısı dik olduğundan;

to that on $B\Gamma$;
 for it is supposed;
 therefore the square on $\Delta\Gamma$
 is equal
 to the square on $B\Gamma$;
 so that the side $\Delta\Gamma$
 to the side $B\Gamma$
 is equal;
 and since equal is ΔA to AB ,
 and common [is] $A\Gamma$,
 the two ΔA and $A\Gamma$
 to the two BA and $A\Gamma$
 are equal;
 and the base ΔA
 to the base $B\Gamma$
 [is] equal;
 therefore the angle $\Delta A\Gamma$
 to the angle $B A\Gamma$
 [is] equal.
 And right [is] $\Delta A\Gamma$;
 right therefore [is] $B A\Gamma$.

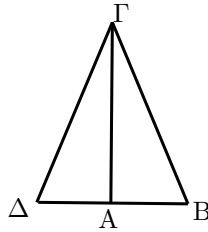
If, therefore, of a triangle,
 the square on one of the sides
 be equal
 to the squares on the remaining two
 sides,
 the angle contained
 by the remaining two sides of the tri-
 angle
 is right;
 —just what it was necessary to show.

τὸ ἀπὸ τῆς $B\Gamma$.
 ὑπόκειται γάρ.
 τὸ ἄρα ἀπὸ τῆς $\Delta\Gamma$ τετραγώνον
 ἴσον ἐστὶ
 τῷ ἀπὸ τῆς $B\Gamma$ τετραγώνῳ.
 ὥστε καὶ πλευρὰ
 ἡ $\Delta\Gamma$ τῆ $B\Gamma$
 ἐστὶν ἴση.
 καὶ ἐπεὶ ἴση ἐστὶν ἡ ΔA τῆ AB ,
 κοινὴ δὲ ἡ $A\Gamma$,
 δύο δὴ αἱ ΔA , $A\Gamma$
 δύο ταῖς BA , $A\Gamma$
 ἴσαι εἰσὶν.
 καὶ βάσις ἡ $\Delta\Gamma$
 βάσει τῆ $B\Gamma$
 ἴση.
 γωνία ἄρα ἡ ὑπὸ $\Delta A\Gamma$
 γωνία τῆ $\text{ὑπὸ } B A\Gamma$
 [ἐστὶν] ἴση.
 ὀρθὴ δὲ ἡ ὑπὸ $\Delta A\Gamma$.
 ὀρθὴ ἄρα καὶ ἡ ὑπὸ $B A\Gamma$.

Ἐὰν ἄρα τριγώνου
 τὸ ἀπὸ μιᾶς τῶν πλευρῶν τετραγώνον
 ἴσον ᾗ
 τοῖς ἀπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
 πλευρῶν τετραγώνοις,
 ἡ περιεχομένη γωνία
 ὑπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
 πλευρῶν
 ὀρθὴ ἐστὶν.
 ὅπερ εἶδει δεῖξαι.

ve BA ile $A\Gamma$ üzerlerindeki kareler eşittirler
 $B\Gamma$ üzerlerindeki kareye;
 çünkü varsayıldı;
 dolayısıyla $\Delta\Gamma$ üzerlerindeki kareye eşittir
 $B\Gamma$ üzerlerindeki kareye;
 böylece $\Delta\Gamma$ kenarı $B\Gamma$ kenarına eşittir;
 ve ΔA , AB kenarına eşit olduğundan, ve $A\Gamma$ ortak, ΔA ve $A\Gamma$ ikilisi BA ve $A\Gamma$ ikilisine eşittirler;
 ve ΔA tabanı $B\Gamma$ tabanına eşittir;
 dolayısıyla $\Delta A\Gamma$ açısı $B A\Gamma$ açısına eşittir.
 Ve $\Delta A\Gamma$ diktir;
 diktir dolayısıyla $B A\Gamma$.

Eğer dolayısıyla bir üçgende bir kenarın üzerindeki kare eşitse üçgenin geriye kalan kenarlarındaki karelere, üçgenin geriye kalan kenarlarınca içeren açı diktir;
 —gösterilmesi gereken tam buydu.



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