## $\Sigma \nu \mu \mu \epsilon \tau \rho i \alpha$ in Aristotle

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The Greek abstract noun  $\sigma \nu \mu \mu \epsilon \tau \rho i \alpha$  is evidently the source of our symmetry. To what extent do the two words share meaning, especially in Aristotle?

The Greek adjective  $\sigma \acute{\nu}\mu\mu\epsilon\tau\rho\sigma_{s}$ ,  $-\alpha$ ,  $-\sigma\nu$  comes to us via Latin as commensurable, and the latter can be used at least sometimes to translate the former. For example, Definition 1 of Book X of Euclid's *Elements* [1, vol. 3, p. 11] is:

Σύμμετρα μεγέθη λέγεται τὰ τῷ αὐτῷ μέτρῳ μετρούμενα, ἀσύμμετρα δέ, ὧν μηδὲν ἐνδεχεται κοινὸν μέτρον γενέσθαι.

Magnitudes measured by the same measure are called **commensurable**; those that admit no common measure, **incommensurable**.

(All translations here are my own.) In the *Metaphysics*, XIII.III.10 (1078<sup>a</sup>35), Aristotle makes a general statement about συμμετρία:

τοῦ δὲ καλοῦ μέγιστα εἴδη τάξις καὶ συμμετρία καὶ τὸ ώρισμένον, ἃ μάλιστα δεικνύουσιν αἱ μαθηματικαὶ ἐπιστῆμαι.

The greatest shapes of the beautiful are arrangement, symmetry, and the delimited, which the mathematical sciences show especially.

(Aristotle's texts can be found at http://www.perseus.tufts.edu. I am using shape here for  $\epsilon l \delta o_s$ , rather than form or species.) It is not clear here whether mathematics is symmetric, or only concerns symmetrical (and arranged, well-defined) things. Aristotle's comment is preceded by:

Since the good and the beautiful are different (for, the former is always in deeds, but the beautiful is also in motionless things), those who say that the mathematical sciences are not about the beautiful and the good are wrong...

The passage does not suggest what symmetry is. Elsewhere in the *Metaphysics*, IV.II.18, (1004<sup>b</sup>11,) we find:

there are particular properties ( $\mathring{l}\delta\iota\alpha \,\pi\acute{a}\theta\eta$ ) of number  $qu\hat{a}$  number, such as oddness/evenness, commensurability/equality, excess/defect...

Here Aristotle seems to be naming properties in correlative pairs. Some of the properties belong to single numbers; others, to pairs. What is the meaning of the middle pair of properties,  $\sigma \nu \mu \mu \epsilon \tau \rho l a / l \sigma \delta \tau \eta s$ ? Possibly a single number can be more or less 'symmetric', depending on how many factors it has. But a single number cannot be 'equal'; equality is a property of pairs of numbers.

Possibly, for Aristotle, every pair of unequal numbers is symmetric in the sense of commensurable: for, the numbers have a common measure, what we call their greatest common divisor. If the numbers are equal, then we would not refer to them as (merely) commensurable; we would say they were equal.

However, the possibility of 'asymmetric' or incommensurable pairs of mathematical objects is suggested at XI.III.7 (1061<sup>a</sup>28):

Just as the mathematician brings about a theory concerning [things obtained] by taking away [i.e. abstraction]—for, he theorizes, having stripped away all that can be sensed, such as weight and lightness...; he leaves only the how-much [i.e. quantity] and the holding-together [i.e. continuity]..., and the properties of things insofar as they are so much and continuous...; for some things, he investigates their placement regarding one another, and what belongs to them; for others, their commensurabilities and incommensurabilities, or their ratios  $(\lambda \acute{o} \gamma oi)$ ...

Symmetry/commensurability in a more practical context arises in the *Nichomachean Ethics* (V.v, 1133<sup>b</sup>16):

Money, as a measure, making [things] commensurable, equalizes  $(i\sigma\acute{a}\zeta\epsilon\iota)$  [them]. For, without commerce, there would be no community;—no commerce, without equality; no equality, without commensurability.

The passage does not make much sense to me unless the verb  $i\sigma a\zeta \omega$  here means something like 'make comparable' rather than 'make equal'; the dictionary suggests also 'balance'. Symmetry in the sense of balance is mentioned in the *Physics* (VII.III, 246<sup>b</sup>4):

We say that all excellences  $(\mathring{a}\rho\epsilon\tau\mathring{a}\iota)$  are in *holding* somehow with respect to something. For, the [excellences] of the body, such as health or vigor, we place in the mixture or balance  $(\sigma\psi\mu\mu\epsilon\tau\rho\iota a)$  of hot things and cold things—these with respect to themselves, or the environment.

(Most passages quoted here are cited in [2] under συμμετρία.)

## References

- [1] Euclid. The thirteen books of Euclid's Elements translated from the text of Heiberg. Vol. I: Introduction and Books I, II. Vol. II: Books III–IX. Vol. III: Books X–XIII and Appendix. Dover Publications Inc., New York, 1956. Translated with introduction and commentary by Thomas L. Heath, 2nd ed.
- [2] Henry George Liddell and Robert Scott. A Greek-English Lexicon. Clarendon Press, Oxford, 1940. Revised and augmented throughout by Sir Henry Stuart Jones.