# Differential fields

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In a differential field, how can we tell whether all consistent systems of equations and inequations have solutions? I shall review the history of answers to this question, and I shall update the accounts in [P1, P2].

To begin with the Robinsonian beginnings, I remind or inform the reader-listener of the following. The class of substructures of models of a theory $T$ is elementary, and its theory is $T_\forall$. The class of structures in which a structure $\mathfrak{M}$ embeds is elementary, and its theory is $\operatorname{diag}(\mathfrak{M})$. The class of models of $T$ is closed under unions of chains if and only if $T = T_{\forall\exists}$ [R2, 3.4.7]. The theory $T$ is called *model-complete* [R1] if $T \cup \operatorname{diag}(\mathfrak{M})$ is complete whenever $\mathfrak{M} \models T$. If $T \subseteq T^*$, and $T_\forall = {T^*}_\forall$, then $T^*$ is the *model-completion* [R2] of $T$ if $T^* \cup \operatorname{diag}(\mathfrak{M})$ is complete whenever $\mathfrak{M} \models T$; but $T^*$ is merely the *model-companion* of $T$ if $T^*$ is model-complete. A *derivation* of a field $K$ is an additive endormorphism $D$ of $K$ that respects the Leibniz rule, $D(x \cdot y) = Dx \cdot y + x \cdot Dy$. A *differential field* is a field equipped with one or more derivations.

## References


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